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Conjugate Gradient Methods in Fitting Precipitation of Rainfall Data in Malaysia

Hua Ru Tang[1], Nur Syarafina Mohamed[2*], Nurul Hajar Mohd Yusoff[3], Norhaslinda Zullpakkal[4]
[1,2]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 39/2, Jalan Ihsan, 80990, Johor Bahru, Malaysia
[3]Fakulti Kejuruteraan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Malaysia
[4]Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Terengganu Branch Kuala Terengganu Campus, Terengganu, Malaysia
**corresponding author: (nursyarafina@utm.my; ORCiD: 0000-0002-8572-2234)*

Abstract – Conjugate gradient method (CGM) is one of the most efficient numerical methods for solving unconstrained optimization problems. It is also known as an iterative method with simple formulation. The classical CGM has always been an interest to the current researchers in improving the formulation which are categorized into three-term (TTCGM), spectral (SCGM), hybrid and scaled CGM. Although there are many variations of the CGM available, choosing the most efficient and effective one for a particular problem can be a time-consuming process. In this study, spectral Hestenes-Stiefel (sHS) CGM with the greatest NOI and central processing time per unit (CPU time) is selected as the efficient method to be applied to the real-life problems in regression analysis. A data set of rainfall precipitation in Malaysia from year 2009 until 2019 is collected for data fitting purpose. The data set is transformed into a test function also defined as objective function. The approximate functions are generated from CG, Least Square, Trendline method for the relative error purpose. The estimation data for the year 2020 can be predicted using the approximate functions. The calculation of relative error of the linear and quadratic model for each method is calculated based on the estimation data for the year 2020 and its actual data. The numerical results show that the sHS CGM is a suitable and good alternative to solve the Least Square models.

Keywords—Spectral Conjugate Gradient Method, Data Estimation, Regression Analysis, Linear Least Square Method, Rainfall Data.

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1. INTRODUCTION

The CGM, originally developed for solving large-scale systems of linear equations in scientific computing, is increasingly relevant in informatics and web engineering due to the growing need for efficient optimization algorithms. In web-based systems, massive datasets often arise from search engine indexing, recommendation systems, or user behaviour modelling. These datasets typically lead to high-dimensional optimization problems where matrix operations are too costly for direct methods. CGM offers an iterative, memory-efficient solution that scales well with

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such applications. Figure 1 shows why CGM was chosen since it needs less iteration to reach the solution point. The CGM is widely utilized across economics, engineering, and scientific disciplines due to its notable advantages [1]. It is commonly employed as an iterative algorithm to obtain numerical solutions [2].

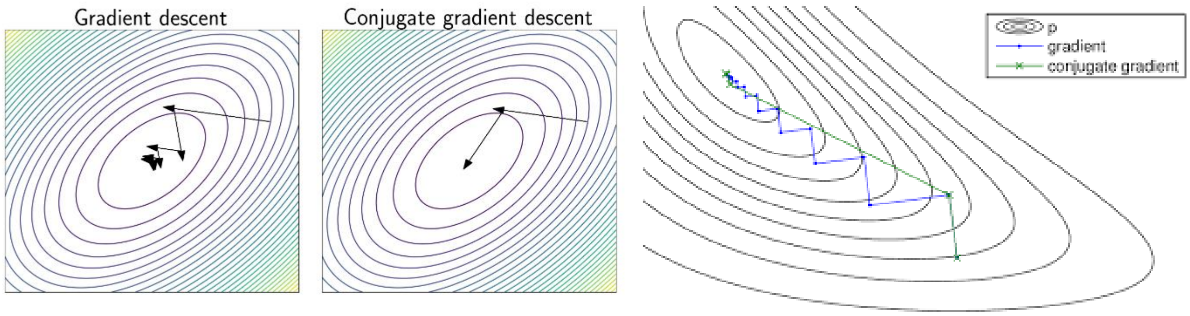

Figure 1. Difference between CGM and Other Gradient Methods

2. LITERATURE REVIEW

The method is typically applied to solve unconstrained optimization problems, which are formulated as follows:

$$\min_{x \epsilon R^n} f(x)$$

$f: R^n \rightarrow R$ is known as continuous differentiable function and R^n represents the n-dimensional Euclidean space. Based on the study by [3], Equation (1) define the sequence of point x_{k+1} start from initial point x_0,

$$x_{k+1} = x_k + \alpha_k d_k, \quad k = 0,1,2, \ldots \tag{1}$$

where x_k denotes the current iteration and $\alpha_k > 0$ indicates the step length obtained from line search. The step length is obtained by any preferable line search either exact or inexact [4], [5]. The exact line search equation is written below

$$\phi(a) = f(x_k + \alpha_k d_k), a > 0$$

The strong Wolfe-Powell (SWP) line search is a type of inexact line search and calculated by

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k$$

$$|g(x_k + \alpha_k d_k)^T d_k| \leq \sigma |g_k^T d_k$$

where $0 < \delta < \sigma < 1$. Different CGMs lead to different search directions. The general search direction d_k in Equation (2) is computed for classical, and the hybrid CGM. The formulation is described as

$$d_k = \begin{cases} -g_k, & k = 0 \\ -g_k + \beta_k d_{k-1}, & k \geq 1 \end{cases} \tag{2}$$

where $g_k = g(x_k)$ and β_k is the CG coefficient. There are five famous CG coefficients by previous researchers such as Hestenes-Steifel (HS) [6], Polak-Ribere-Polyak (PRP)) [7], Fletcher-Reeves (FR) [8], Liu-Storey (LS) [9] and Dai-Yuan (DY) [10]. Some of these classical CGMs are improved by combining two classical CGMs which are called hybrid CGMs. The examples of hybrid CGMs are HS-PRP [11], HS-LS [12], LS-DY [13] and FR-DY [14]. The improvement by previous researchers is not limited to the CG coefficient only where the search direction formulas are improved too. The modifications of CGM have been done vigorously since there are a lot of CG types that can be modified – TTCGM and SCGM. The search direction of TTCGM and SCGM consists of another parameter in its formulation. The TTCGM has three terms in its formulation which is written as in Equation (3)

$$d_k = \begin{cases} -g_k, & k = 0 \\ -g_k + \beta_k d_{k-1} - \theta(g_k - g_{k-1}), & k \geq 1 \end{cases} \tag{3}$$

where β_k is the CG coefficient and θ_k is parameter. The examples of TTCGM are introduced by [15], [16], [17].

Equation (4) shows the search direction of the SCGM which is defined as

$$d_k = \begin{cases} -g_k, & k = 0 \\ \beta_k d_{k-1} - \theta_k g_k, & k \geq 1 \end{cases} \quad (4)$$

and β_k refers to CG coefficient and θ_k is spectral parameter.

This study focuses on SCGM, which has been widely studied by recent researchers in solving real world problems. Implementing the CGM within the platform's backend optimization pipeline enables - Reduced computation time, lower memory footprint and scalable deployment. [18] introduced a free-derivative SCGM to tackle nonlinear monotone equation systems with convex constraint. [19] modified Dai-Liao SCGM to solve signal processing problem. [20] implemented two sufficient descent SCGM in image restoration problems, as shown in Figure 2.

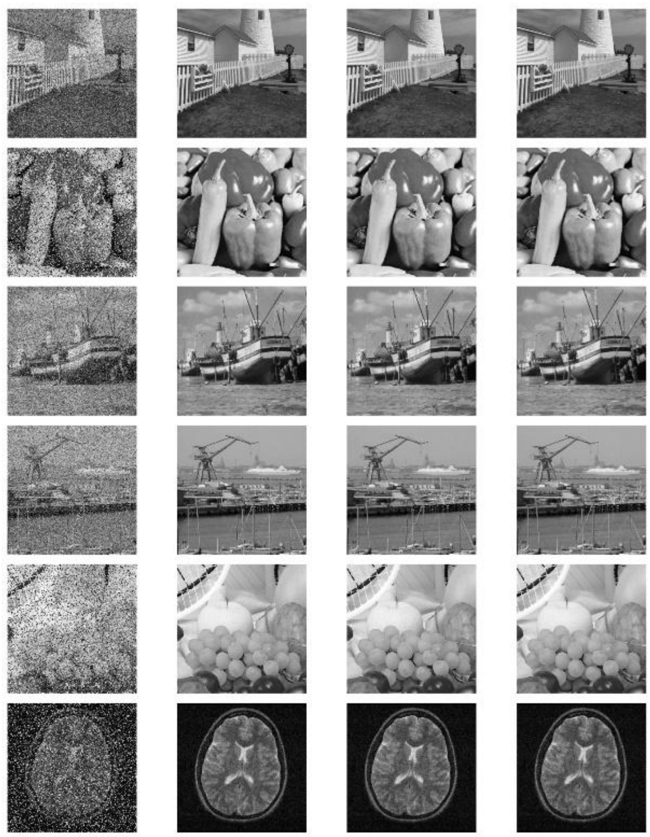

Figure 2. An Application of CGM in Image Restoration [20]

For application, this study focuses on applying the SCGM chosen to estimate the rainfall data and compare to other estimator tools. Rainfall is the result of atmospheric moisture falling to the Earth's surface due to gravity. It occurs when part of the atmosphere becomes saturated with water vapor, leading to condensation and the formation of precipitation. There are various forms of precipitation, such as rain, snow, hail, sleet and drizzle. In Malaysia, the only form for the precipitation is rainfall. Due to its uncontrollable and unpredictable, predicting rainfall data is crucial in reducing operational costs [21]. As it can maintain water levels for electricity generation, but it can also cause floods that threaten human lives. According to [22], [23], rainfall forecasts are commonly used to issue flash flood warnings earlier in an event, providing more time for preparation than relying on observations alone.

The CGM has been applied to data estimation. Basically, this method is used to fit rainfall data to form mathematical models and make predictions about future rainfall patterns. With low data storage, efficiency and convergence speed of the classical CGM, SCGM is selected to solve this problem. A regression model is generated and computed from the best SCGM to estimate the precipitation of the rainfall data in Malaysia.

3. RESEARCH METHODOLOGY

In this paper, the method used is CGM specifically on SCGM. SCGM is chosen as there have been modifications to that type, considering its simplicity and better convergence. The selection of SCGM is motivated by its advantages in solving unconstrained nonlinear optimization problems, particularly its robustness in terms of convergence behaviour and computational efficiency. The sHS variant was chosen due to its proven performance in minimizing objective functions with fewer iterations NOI and reduced CPU time, which are critical factors in large-scale environmental data modelling.

3.1 Spectral Conjugate Gradient Method (SCGM)

The SCGM is known as a two-term, SCGM which involves CG coefficient and parameter of θ_k . Recently, there were many types of modified SCGM but in this study only four latest SCGM are chosen as shown in Equations (5) – (8). [24] proposed for spectral Rivaie, Mustafa, Ismail and Leong and it is denoted as sRMIL. This method is defined as

$$\beta_k^{sRMIL} = \frac{g_k^T(g_k - g_{k-1})||d_{k-1}||^2}{||d_{k-1}||^2}, \theta_k = 1 - \frac{g_k^T d_{k-1}}{g_{k-1}^T d_{k-1}} \tag{5}$$

[25] studied a spectral Fletcher-Reeves (sFR) CGM where the parameters θ_k and β_k are defined as

$$\beta_k^{sFR} = \frac{||g_k||^2}{||g_{k-1}||^2}, \theta_k = \frac{d_{k-1}^T (g_k - g_{k-1})}{||g_{k-1}||^2} \tag{6}$$

[26] proposed the spectral Conjugate Descent (sCD) which is given by

$$\beta_k^{sCD} = -\frac{||g_k||^2}{d_{k-1}^T g_{k-1}}, \theta_k = 1 - \frac{g_k^T d_{k-1}}{g_{k-1}^T d_{k-1}} \tag{7}$$

[27] proposed the sHS parameter is proposed by

$$\beta_k^{sHS} = \frac{g_k^T(g_k - g_{k-1})}{d_{k-1}^T(g_k - g_{k-1})}, \theta_k = 1 - \frac{g_k^T g_{k-1} g_k^T d_{k-1}}{||g_k||^2 d_{k-1}^T(g_k - g_{k-1})} \tag{8}$$

These selected SCGM are going to be tested on different number of test functions to evaluate their performance. The performance is analysed by the performance ration based on number of iterations (NOI) and CPU times. The least numbers of NOI and CPU give the best methods in terms of their efficiency. The algorithm of SCGM is shown in Algorithm 1.

Algorithm 1: Algorithm of SCGM

Based on the study by [28], the computation of algorithm process
Step 1 : Initialization process and generate x_0 starting with $k = 0$.
Step 2 : Compute the parameteβ$_k$ r based on (4), (5), (6) and (7).
Step 3 : Compute search directions based on equation (3), if$||g_k|| = 0$, stop.
Step 4 : Compute step size, α_k by line searches.
Step 5 : Set $x_{k+1} = x_k + \alpha_k d_k$. Then, define β_{k+1} and compute θ_{k+1} satisfying θ_k parameter.
Step 6 : Set $k = k + 1$, go to Step 3.

Once the most superior method is chosen, the method is then implemented to the rainfall dataset to estimate the data. The validity of the results is compared to the existing methods used in data estimation which is the least square method in linear regression analysis.

3.2 Linear Regression

The implementation of the best SCGM coefficient is tested by using Least Square method of regression analysis. The best method of SCGM is selected to fit the precipitation rainfall data in Malaysia. The linear regression model is generated based on the data collected. The linear regression model formula represented by $\mathrm{y} = \mathrm{a}_0 + \mathrm{a}_1\mathrm{x}_1 + \mathrm{a}_2\mathrm{x}_2 + \cdots + \mathrm{a}_\mathrm{p}\mathrm{x}_\mathrm{p} + \epsilon$ where $\mathrm{a}_0, \mathrm{a}_1, \mathrm{a}_2, \ldots., \mathrm{a}_\mathrm{p}$ is the regression parameters. This formula forms a linear equation that can fit the data better.

3.3 Least Square Method

The relative error is calculated for the Least Square method purpose. The Least Squares method determines the optimal model by finding the difference between predicted and actual values. This error is calculated by comparing the estimated values to the observed data as shown in Equation (9).

$$\text{relative error} = \frac{|\text{exact value} - \text{approximate value}|}{|\text{exact value}|} \tag{9}$$

The formula to minimize the sum of the residual error squares for the data given by Equations (10) and (11),

$$Min \sum_{i=1}^{m} E_i^2 = \sum_{i=1}^{m} \left((a_0 + a_1 x) - y_i \right)^2 \tag{10}$$

$$Min \sum_{i=1}^{m} E_i^2 = \sum_{i=1}^{m} \left((a_0 + a_1 x + a_2 x^2) - y_i \right)^2 \tag{11}$$

The formula to obtain the values of a_0 and a_1 for a linear model is shown in Equation (12).

$$\begin{bmatrix} m & \sum_{i=1}^{m} E_i^2 \\ \sum_{i=1}^{m} E_i^2 & \sum_{i=1}^{m} E_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{m} y_i \\ \sum_{i=1}^{m} x_i y_i \end{bmatrix} \tag{12}$$

The formula for finding values of a_0, a_1 and a_2 for a quadratic model is in Equation (13).

$$\begin{bmatrix} m & \sum_{i=1}^{m} x_i & \sum_{i=1}^{m} x_i^2 \\ \sum_{i=1}^{m} x_i & \sum_{i=1}^{m} x_i^2 & \sum_{i=1}^{m} x_i^3 \\ \sum_{i=1}^{m} x_i^2 & \sum_{i=1}^{m} x_i^3 & \sum_{i=1}^{m} x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{m} x_i \\ \sum_{i=1}^{m} x_i\, y_i \\ \sum_{i=1}^{m} x_i^2\, y_i \end{bmatrix} \tag{13}$$

The functions of linear and quadratic models are given by Equations (14) and (15),

$$f(x) = a_0 + a_1 x \tag{14}$$

$$f(x) = a_0 + a_1 x + a_2 x^2 \tag{15}$$

respectively. The value of a_0, a_1 and a_2 are obtained by formula in Equations (12) and (13).

Algorithm 2: Algorithm of Least Square method

Step 1 : Identify the linear and quadratic formula from (12) and (13).
Step 2 : Identify variables and the data calculation based on (12) and (13).
Step 3 : Compute the value of a_0, a_1 and a_2
Step 4 : Formulate the approximate functions for linear and quadratic using real data.
Step 5 : Find the error using by (9), (10) and (11).
Step 6 : Model is estimated.

From (10), (11), (12) and (13), Least Square method can transform into optimization problems for linear, Equation (16) and quadratic, Equation (17) model respectively as follows

$$\min_{x \epsilon R^n} f(x) = \sum_{i=1}^{m} \big((a_0 + a_1 x) - y_i\big)^2 \tag{16}$$

$$\min_{x \epsilon R^n} f(x) = \sum_{i=1}^{m} \big((a_0 + a_1 x + a_2 x^2) - y_i\big)^2 \tag{17}$$

Algorithm 3: Least Square Conjugate Gradient Algorithm

Step 1 : Initialization. Let $k = 0$, select $x_0 = (a_o, a_1)$ for linear and $x_0 = (a_o, a_1, a_2)$ for quadratic. Solve (16) and (17).
Step 2 : Compute β_k for sHS.
Step 3 : If $||g_k|| < \epsilon$, stop. Else, calculate the search direction d_k by (3).
Step 4 : Solve α_k using line searches.
Step 5 : Update new iterative point by using (1) and substitute the updated point in (14) and (15).
Step 6 : If $f(x_{k+1}) < f(x_k)$ and $||g_k|| < \epsilon$, stop. Otherwise, go to Step 2 with $k = k + 1$.

3.4 Excel Trend Line Method

The values of a_0 and a_1 also can be determined by the Excel Trend Line Method. A linear graph of precipitation of rainfall data versus months can be generated by this method. The linear regression model is generated automatically. Then, the comparison of real value and approximation value can be obtained by computing the relative error.

4. RESULTS AND DISCUSSIONS

Numerical experiments are done to test the method robustness and efficiencies. Ten optimization test functions of large-scale unconstrained optimization are selected from [29] to compute the methods chosen in equations (5) – (8). The experiment was done by implementing the coding to MATLAB program to test the NOI and CPU times for each method chosen. The numerical comparison among the SCGM (sRMIL, sFR, sCDand sHS) are then recorded based on NOI and CPU times using the performance profile.

A set of test functions from Table 1 are solved by the four tested SCGM under strong Wolfe line search using MatlabR2023a subroutine programming. The numerical result of central processing unit (CPU) time and NOI of each spectral method are recorded to find the most efficient and robust method.

The results obtained are interpreted into the performance profile as in Figure 3 and Figure 4. The $P_s(\tau)$ of performance profile represents the fraction of problems with a ratio parameter τ. The higher the value of $P_s(\tau)$, the higher the efficiency of the solver. The most efficient SCGM represented the top curve in the graph.

Table 1. List of Test Function

No	Test Functions	Variable	Initial Points
1	Six Hump	2	(2, 2), (5, 5), (12, 12), (20, 20)
2	Zettl	2	(2, 2), (6, 6), (12, 12), (25, 25)
3	Trecanni	2	(4, 4), (8, 8), (10, 10), (15, 15)
4	Booth	2	(2, 2), (10, 10), (18, 18), (50, 50)
5	Generalized Quartic	2	(2, 2), (12, 12), (25, 25), (32, 32)
		4	(2, …, 2), (5, …, 5), (10, …, 10), (18, …, 18)
		10	(2, …, 2), (5, …, 5), (10, …, 10), (25, …, 25)
6	Extended Tridiagonal 1	2	(2, …, 2), (8, …, 8), (12, …, 12), (20, …, 20)
		4	(2, …, 2), (4, …, 4), (10, …, 10), (25, …, 25)
		10	(2, …, 2), (4, …, 4), (10, …, 10), (25, …, 25)
7	FLETCHCR	2	(2, …, 2), (5, …, 5), (12, …, 12), (25, …, 25)
		4	(2, …, 2), (4, …, 4), (15, …, 15), (25, …, 25)
		10	(2, …, 2), (4, …, 4), (10, …, 10), (20, …, 20)
8	Diagonal 4	2	(2, …, 2), (4, …, 4), (10, …, 10), (25, …, 25)
		4	(2, …, 2), (5, …, 5), (10, …, 10), (25, …, 25)
		10	(2, …, 2), (4, …, 4), (10, …, 10), (25, …, 25)
		500	(2, …, 2), (4, …, 4), (10, …, 10), (25, …, 25)
		1000	(2, …, 2), (4, …, 4), (10, …, 10), (25, …, 25)
9	Extended Himmelblau	2,4,10, 500, 1000	(2, …, 2), (4, …, 4), (10, …, 10), (25, …, 25)
10	Extended Rosenbrock	2,4,10, 500, 1000	(2, …, 2), (4, …, 4), (10, …, 10), (25, …, 25)

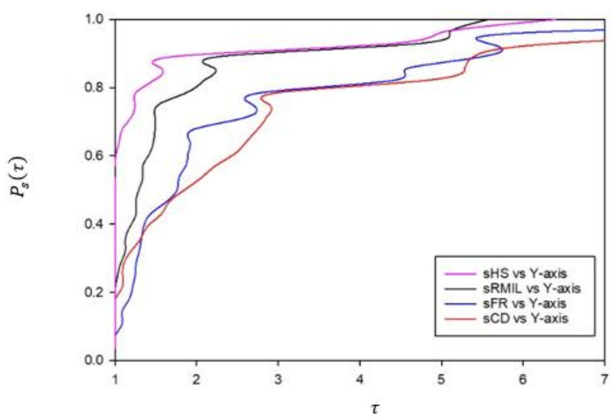

Figure 3. Performance Profile Based on CPU Time

The performance profile shows the performance ratio of all the tested solvers. The top left curve indicates the highest amount of test functions solved with the best NOI or CPU time. The top right curve indicates the amount of test function successfully solved. From the performance profile on the CPU time in Figure 3, the sRMIL method has the best efficiency followed by sHS, sCD and sFR but from the performance profile on NOI in Figure 4 shows that the sHS has the best efficiency followed by sRMIL, sCD and sFR. By comparing both performance profiles, even though the sRMIL method is more efficient than sHS in CPU time, the sHS method obviously has higher efficiency compared to sRMIL in term of NOI. Therefore, the sHS method is the best solver performance for the SCGM in this study.

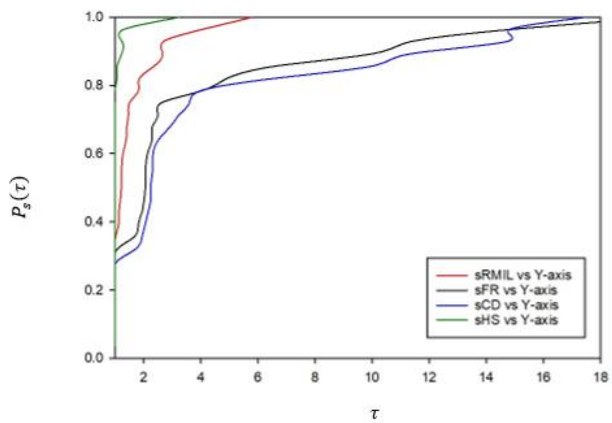

Figure 4. Performance Profile Based on NOI

4.1 Application on Rainfall Precipitation in Malaysia

The implementation of the sHS CG coefficient can be done by using the Least Square method of regression analysis. The aim of the implementation is to show that the sHS CGM can be applied for analyzing real data in statistics. The precipitation of rainfall in Malaysia is estimated by using Least Square method, Least Square CGM and Excel Trend Line method for linear and quadratic models [30]. The collected data of the precipitation of rainfall in Malaysia for 12 years (2009 to 2020) are tabulated in Table 2. This data is collected from the World Bank. The data in unit millimeters is the total amount of precipitation from January to December of each year. The numbers of data in Table 2 denoted as variable while the precipitation (mm) denoted as variable. The data from the year 2009 to 2019 are selected for the data fitting process. The data for the year 2020 is used for error calculation.

Table 2. Precipitation of Rainfall in Malaysia from 2009 to 2020

Number of data	Years	Precipitation (mm)
1	2009	3404.89
2	2010	3104.02
3	2011	3444.40
4	2012	3222.64
5	2013	3095.05
6	2014	2808.69
7	2015	2661.85
8	2016	2829.83
9	2017	3262.68
10	2018	3242.89
11	2019	2598.72
12	2020	3054.01

Let x_i be the number of years and y_i be the precipitation of rainfall in Malaysia. The linear and quadratic models are formulated using collected data from the year 2009 to 2019. The calculation of the data needed based on the Least Square formula in Equations (12) and (13) are given as follows.

$$\begin{bmatrix} 11 & 66 \\ 66 & 506 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 33675.7 \\ 196814.6 \end{bmatrix}$$

and

$$\begin{bmatrix} 11 & 66 & 506 \\ 66 & 506 & 4356 \\ 506 & 4356 & 39974 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 33675.7 \\ 196814.6 \\ 1491423 \end{bmatrix}$$

The values of a_0, a_1 and a_2 can be obtained by expanding and solving the above matric expression. The value of a_0, a_1 and a_2 calculated is then substitute into the formula (14) for linear and (15) for quadratic model. The approximate functions for both Least Square models are as below

Linear Least Square model:

$f(x) = 3347.223636 - 47.63272727272733x$

Quadratic Least Square model:

$f(x) = 3505.28424242424 - 120.583776223776x + 6.07925407925408x^2$

For Least Square CGM, the first eleventh years (2009 – 2019) in Table 2 are used to form linear and quadratic optimization problems in (16) and (17). The linear and quadratic models are generated using MATLAB and the functions obtained for both optimization model is as below

$f(a_0, a_1) = 11{a_0}^2 + 132a_0a_1 - 6.7351(10^4)a_0 + 506{a_1}^2 - 3.9363(10^5)a_1 + 1.0395(10^8)$

$$f(a_0, a_1, a_2) = 11{a_0}^2 + 132a_0a_1 + 1012a_0a_2 - 6.7351(10^4)a_0 + 506{a_1}^2 + 8712a_1a_2 - 3.9363(10^5)a_1 + 39974{a_2}^2 - 2.9828(10^6)a_2 + 1.0395(10^8)$$

By setting the above optimization models as the test function and solved by the Least Square Conjugate Gradient Algorithm. The solution point for the value of a_0, a_1 and a_2 can be obtained by using any initial point. The result of linear and quadratic model for sHS CGM are given by

Linear sHS CG model:

$f(x) = 3347.202727273334 - 47.629999995020x$

Quadratic sHS CG model:

$f(x) = 3504.526969699340 - 120.241188825254x + 6.050932402436x^2$

For Excel Trend Line method, the values of a_0 and a_1 also can be determined by the Excel Trend Line Method. A linear graph of precipitation of rainfall data versus year can be generated by this method. The linear regression model will be generated automatically. The linear and quadratic trend line are computed from the data from Table 2. The trend line indicates the best fit line. The graph based on the equations in (14) and (15) are generated by using Microsoft Excel software. The models generated are

Linear Trend Line model:

$f(x) = 3347.2063636363600 - 47.6304545454542x$

Quadratic Trend Line model:

$f(x) = 3505.22787878782000 - 120.56346153845500x + 6.07775058275115x^2$

The graphs of the linear and quadratic models are shown in Figures 5 and 6.

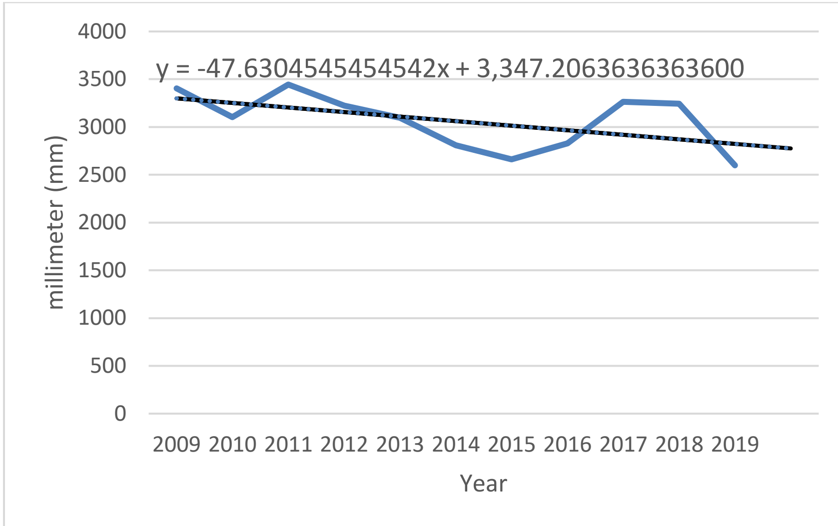

Figure 5. Linear Trend Line for Precipitation of Rainfall in Malaysia

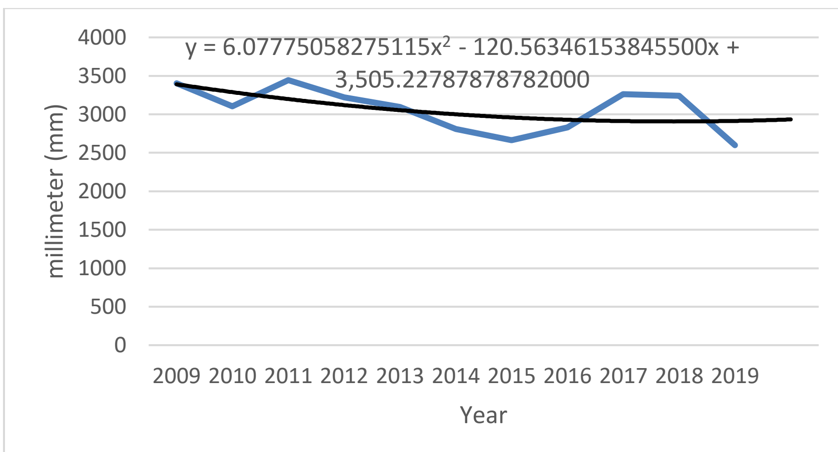

Figure 6. Quadratic Trend Line for Precipitation of Rainfall in Malaysia

The efficiency of a method can be identified by the relative absolute error. The data for the year 2020 is chosen to be estimated by each model i.e when $x=12$. By comparing the actual and estimated data, the relative errors are calculated from each of the models by formula in Equation (9). Table 3 shows the calculation of the relative error for each model.

Table 3. Estimation Values and Relative Errors

Methods	Estimation Values	Relative Error
Linear Least Square	2775.63090872727	0.0911519907507606
Quadratic Least Square	2933.69151515152	0.0393968863391018
Linear sHS CG	2775.64272733309	0.0911481208859533
Quadratic sHS CG	2932.96696974708	0.0396341302919507
Linear Trend Line	2775.64090909091	0.0911487162481754
Quadratic Trend Line	2933.66242424253	0.0394064118183864

Based on Table 3, the relative error of the Quadratic sHS CGM is smaller than Linear Least Square, Linear sHS CGM and Linear Trend Line method, but it is approximately similar compared to the relative error of Quadratic Least Square and Quadratic Trend Line Method. The best method is the Quadratic Least Square Method with the smallest relative error compared to other methods. Although the relative error difference between the SCGM approach and the conventional statistical method may appear relatively small, it is important to note that statistical methods are traditionally and widely used for data estimation in many fields. However, the results from this study highlight the potential of SCGM, particularly the sHS variant, as a reliable and competitive alternative for data estimation tasks. Beyond the comparable accuracy, SCGM demonstrates superior computational efficiency, as evidenced by its faster convergence rate in achieving the lowest NOI and reduced CPU processing time during performance evaluations. These advantages indicate that SCGM is not only capable of providing accurate estimations but also offers a more efficient computational solution, making it highly suitable for large-scale or real-time forecasting applications.

5. CONCLUSION

In conclusion, the SCGM method, particularly the sHS method demonstrates strong performance, making it a robust approach due to its ability to achieve the lowest NOI and the shortest CPU time. The application of the sHS method in data estimation demonstrates that the quadratic model, combined with the strong Wolfe line search, is highly effective for estimating rainfall precipitation in Malaysia. This model surpasses the linear approach by producing a lower relative error. The estimated rainfall value generated is 2932.97 mm with a relative error of 0.0396. Although the difference in relative error compared to the statistical method which was commonly used for data estimation may appear minimal, the results clearly show that SCGM is a superior and effective data estimation method. This is further supported by its superior convergence performance, as evidenced by the lowest NOI and reduced CPU time during testing.

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AUTHOR CONTRIBUTIONS

Hua Ru Tang: Conceptualization, Data Curation, Methodology, Validation, Writing – Original Draft Preparation;
Nur Syarafina Mohamed: Project Administration, Supervision, Writing – Review & Editing;
Nurul Hajar Mohd Yusoff: Language Assessor– Review & Editing;
Norhaslinda Zullpakkal: Project Administration, Supervision, Writing – Review & Editing.

CONFLICT OF INTERESTS

No conflict of interest was disclosed.

ETHICS STATEMENTS

The data used in this research collected from World Bank Open Data, which is an open-source and freely available website responsible for cataloging, visualizing, and analyzing data from https://data.worldbank.org/indicator/AG.LND.PRCP.MM. Our publication ethics follow The Committee of Publication Ethics (COPE) guideline. https://publicationethics.org/.

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BIOGRAPHIES OF AUTHORS

	**Hua Ru Tang** did her bachelor's degree in Department of Mathematical Science at Universiti Teknologi Malaysia (UTM). Her academic interests focus on operational research. This paper reflects her commitment to exploring innovative ideas and contributing to scholarly discussions within the field. She can be contacted at email: tanghuaru@graduate.utm.my.
	**Nur Syarafina Mohamed** is a Senior Lecturer at the Department of Mathematical Sciences, Universiti Teknologi Malaysia (UTM). She obtained her PhD in Mathematics from Universiti Sultan Zainal Abidin (UNISZA) in 2017, after completing her MSc and BSc in Mathematics at UTM. Her research focuses on optimization and operational research, particularly in mathematical modelling, image processing, and data analysis. With extensive publications, she actively advances both theoretical studies and practical solutions, contributing significantly to the development of mathematical sciences. She can be contacted at email: nursyarafina@utm.my.
	**Nurul Hajar Mohd Yussoff** is a lecturer at Universiti Teknikal Malaysia Melaka (UTeM) with a PhD in Mathematics (Optimization) from Universiti Sultan Zainal Abidin. Her expertise lies in optimization methods, particularly conjugate gradient algorithms, with applications in numerical analysis, engineering, data modelling, and emerging technologies. She has published extensively in mathematical optimization and contributes to interdisciplinary projects. She can be contacted at email: nurulhajar@utem.edu.my.
	**Norhaslinda Zullpakkal** is a Senior Lecturer in the Faculty of Computer and Mathematical Sciences at UiTM Cawangan Terengganu Kampus Kuala Terengganu. Her research primarily focuses on optimization, with special expertise in conjugate gradient methods and their hybrid variants. She has authored several notable papers, including work on the global convergence of new spectral CGMs and the application of CGMs for COVID-19 data modelling and image restoration problems. She can be contacted at email: lindazullpakkal@uitm.edu.my.