Vol 6 No 1 (2024) *e*-ISSN: 2682-8383

Journal of Engineering Technology and Applied Physics

The Review of Recent Trend for School Bus Routing Problem

Nur Intan Liyana binti Mohd Azmi*, Afwan Hakim Bin Md Mahdzir, Heng Yu Xuan, Ng Xian Yu, Tan Mei Tiean and Tian Yu Kun

Faculty of Science, Universiti Tunku Abdul Rahman, Kampar, Perak, Malaysia.

**Corresponding author*: intanliyana@utar.edu.my, *ORCiD*: 0009-0002-3739-6208

https://doi.org/10.33093/jetap.2024.6.1.9

Manuscript Received: 14 August 2023, Accepted: 18 November 2023, Published: 15 March 2024

***Abstract* — The School Bus Routing Problem (SBRP) is a complex transportation challenge involving finding optimal bus routes. This review paper provides an overview of the recent developments in SBRP research and focuses on three sub-problems: Bus Route Generation, Bus Route Scheduling, and Bus Stop Selection. The paper examines recent publications from 57 relevant articles. It highlights the increasing focus on real-world and complex scenarios, as well as the growing popularity of metaheuristic approaches in addressing SBRP challenges. The analysis reveals the significance of bus route generation, bus route scheduling, and bus stop selection, showcasing the effectiveness of machine learning and heuristic or metaheuristic algorithms in improving route quality. This study also classifies SBRP problems based on the number of schools, service surroundings (urban or rural), mixed-load scenarios, and fleet mix (homogeneous or heterogeneous). Finally, the paper explores the objectives of SBRP research, including minimising the total cost, distance, time, and number of buses. Meanwhile, the constraints of this study are the capacity of a bus, the maximum riding time, time windows, the maximum walking time between two stops and so on. This comprehensive review paper aims to offer a framework for new researchers and provides valuable insights for future research directions in this transportation area.**

Keywords—School Bus Routing Problem (SBRP), Exact and heuristic algorithm, GAMS software

I. INTRODUCTION

The School Bus Routing Problem (SBRP) is a multifaceted transportation challenge involving finding optimal routes for school buses. It has been an active area of research since 1969 by Newton and Thomas [1]. SBRP is a complex problem with various sub-problems, each of which can be treated as a distinct optimisation problem. For example, the Vehicle Routing Problem (VRP) and generating bus routes are interlinked sub-problems [2]. Taking this approach allows us to tackle specific components of the problem with targeted optimisation techniques. An effective SBRP solution improves service quality and reduces costs [3].

An updated review of the School Bus Routing Problems (SBRP) literature is needed due to increased publications and changes in research focus. Recent studies in SBRP have focused on practical concerns like serving multiple schools, mixed loading, and heterogeneous fleets [4]. These factors enhance the models' usefulness but also make finding solutions more challenging. Metaheuristic solution techniques are increasingly used to address these complex SBRP scenarios. Hence, this report provides an overview of the current research developments in SBRP.

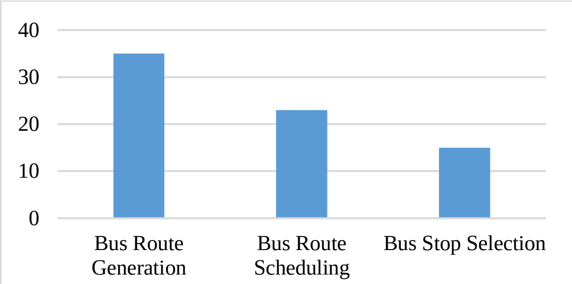

Fig. 1. Frequency of SBRP's sub-problems.

This review of SBRP literature involved searching Google Scholar with relevant keywords, resulting in evaluating 48 papers, 40 of which were recent. The review focused on three sub-problems: Bus Route Generation, Bus Route Scheduling, and Bus Stop Selection. These sub-problems are often studied independently. A graphical representation (Fig. 1) illustrates the frequency of references to these sub-problems between 2018 and 2023, indicating their significance in SBRP research. Table I provides a comprehensive overview of the research in this field.

Published by MMU PRESS. URL: https://journals.mmupress.com/index.php/jetap/index

Table I: Description of SBRP past references.

References	Sub-problem Types			Number of Schools		Environment		Mixed-load		Fleet Mix	
	BRG	***BRS***	***BSS***	***S***	***M***	***U***	***R***	***Yes***	***No***	***HO***	***HT***
Jaradat and Shatnawi (2020)	√	√				√					
Oluwadare, Oguntuyi and Nwaiwu (2018)	√			√					√	√	
Ellegood *et al.* (2020)	√	√	√		√	√	√	√			√
Noor *et al.* (2020)		√	√	√		√		√			
Ali Shafahi *et al.* (2018a)	√	√	√	√		√		√		√	
Armond *et al.* (2022)	√									√	
Han and Zhang (2019)	√				√	√			√	√	
Guo *et al.* (2019)	√					√		√		√	
Ozmen and Sahin (2021)	√				√	√		√		√	
Banerjee and Smilowitz (2019)		√			√	√			√		
Sciortino (2022)	√	√	√	√					√		√
Bertsimas *et al.* (2019)	√				√			√			√
Ochoa-Zezzatti et al. (2020)	√	√									
Ren *et al.* (2019)	√	√	√		√	√		√		√	
Li and Chow (2021)	√	√	√		√	√		√			√
Shang *et al.* (2021)	√		√		√			√			√
Ansari *et al.* (2021)	√	√	√	√				√			
Miranda *et al.* (2021)	√	√	√			√	√	√			√
Mokhtari and Ghezavati (2018)	√		√	√			√	√			√
Komijan *et al.* (2021)	√	√			√	√	√	√		√	
Miranda *et al.* (2018)	√		√		√		√	√			√
Ali Shafahi *et al.* (2018b)		√	√	√		√	√	√		√	
Ümit *et al.* (2019)	√	√				√	√	√			√
Prah *et al.* (2018)	√	√		√		√	√	√			√
Sánchez-Ansola *et al.* (2022)	√							√			√
Calvete *et al.* (2020)	√	√			√	√	√	√			√
Lewis *et al.* (2018)	√			√		√			√	√	
Hou *et al.* (2022)	√			√		√		√			√
Guo *et al.* (2022)	√	√		√		√			√	√	
Dang *et al.* (2019)	√			√		√			√	√	
Calvete *et al.* (2022)	√					√		√		√	
Hou *et al.* (2022)	√	√		√		√		√			√
Sales *et al.* (2018)	√	√		√		√		√			√

This study aims to summarise recent advances in SBRP research by categorising existing literature and evaluating key components. It also considers older publications published for a comprehensive understanding of the solutions used. The article introduces problem characteristics and aspects for achieving research goals. It further delves into SBRP research by concentrating on real-world and intricate scenarios. The study aims to assist new scholars in the field by offering typical solution techniques and future research directions.

II. SUB-PROBLEMS

A. *Bus Route Generation*

SBRP considers the creation of effective school bus routes to be a crucial sub-problem. As shown in Fig.1, 30 of the 33 publications from the most recent five years in this evaluation specifically address bus route generation. Recent studies show a growing interest in innovative approaches like machine learning and heuristic/metaheuristic algorithms for bus route generation. For instance, applied machine learning techniques [5], including Artificial Neural Network (ANN) and Support Vector Machine (SVM), to predict shuttle bus trip time, fuel consumption, and emissions. They highlighted the value of sustainability and the potential for machine learning to enhance transportation management. While [6] supports the concept of a hybrid algorithm combining Genetic Algorithm (GA) and Simulated Annealing Algorithm (SAA) to achieve a feasible solution for route generation. [7] proposed a two-step heuristic approach integrating schedule information, leading to a 25% improvement in existing solutions to benchmark problems. [8] introduced an Ant Colony Optimization (ACO) based algorithm that generates high-quality bus routes for commuters. [9] developed an enhanced ant colony method considering road traffic and service quality to save costs and improve service quality. Additionally, Genetic Algorithm (GA) approaches have been proposed to minimise travel time and enhance the quality of service (e.g., [2, 10, 11, 12]). These studies demonstrate the significance of bus route generation and highlight the effectiveness of machine learning and heuristic/metaheuristic algorithms in improving route quality.

B. *Bus Route Scheduling*

One of the critical sub-problems in the SBRP is the bus route schedule [4] investigated that bus route scheduling involves a temporal dimension and considers variables like travel time between stops and loading time at each stop. Planning bus routes must consider both the time spent between stops and the time spent loading at each stop proposed an approach to tackle the School Bus Scheduling Problem (SBSP) [13], aiming to decrease the number of buses needed by a school district. The SBSP involves deciding on school bell times and scheduling bus routes accordingly. The authors conducted numerical experiments to validate their models and offered suggestions for incorporating equity in changes to school start times. Furthermore, [12] developed a bi-objective optimization method that enhances the efficiency of the genetic algorithm for scheduling school buses and [14] suggested a solution that allows students to switch buses within a single trip and handle multiple trips for different schools within specific time slots. These studies offer valuable insights into improving the effectiveness of school bus transportation by addressing the bus route scheduling problem.

C. Bus Stop Selection

The objective of this sub-problem is to determine where each student is supposed to wait for the bus based on a list of possible places that the school district has approved. A study by [4] and [15] focuses primarily on the bus stop selection problem and aims to choose a subset of bus stops from a list of probable bus stops and allocate students to these stops. On the other hand, most of the current SBRP research has focused on this particular aspect of the problem [4].

On the other hand, some writers are more concerned about researching a solution for selecting school bus stops related to SBRP [16, 17]. For instance, [16] presents a novel optimisation model for the SBRP that minimises the overall number of stops. This model is based on a school bus routing technique referred to as bi-objective routing decomposition (BiRD), and it uses this approach to determine the optimal route. In contrast, [17] presented a Java library that would identify the ideal route for school bus routing by utilising the technique of a near neighbour to discover the optimal route that traverses all bus stops.

Some researchers have emphasised the simultaneous development of bus routes and the selection of bus stops as a potential solution to optimise school bus difficulties [4, 18]. A reinforcement learning-enabled genetic algorithm for school bus scheduling concurrently by [12, 19]. This approach handles the critical issue of bus stop selection, which has been a problem for a long time. To further optimise the location and routing of bus stops, [12] used the LAR (Location-Allocation-Routing) strategy, which takes into account the relative position of the pick-up point and its nearest road segment while [19] uses a combination of LAR and ARL (Allocation-Routing-Location) approaches to ensure that all students are able to reach them conveniently.

In a recent study, [20] have formulated a basic set to cover the problem:

$$\text{Min } Z = \sum_{j \in N} X_j^G \tag{1}$$

subject to

$$\sum_{j \in N_i} X_j^G \geq 1, \forall i \in G \tag{2}$$

$$X_j^G \in \{0,1\}, \forall j \in N \tag{3}$$

, where
N = set of potential locations of GE (General Education) stops, where $N_i \subseteq N$ is the subset of the potential GE stops that can cover node I with a distance d_{ij} within the maximum distance D
$N_i = \{j \mid d_{ij} \leq D\}$
G = set of GE students' locations
X_j^G = binary variable is 1 if a GE stop is established at point j∈N, 0 otherwise
Objective function 1 minimises the number of GE

III. CLASSIFICATION OF PROBLEMS

A. Number of Schools (Single or Multiple)

The single school defines a scenario of looking at school districts with several schools but employing a technique that only considers one school at a time. However, multiple schools indicate a scenario in which students from different schools are grouped together by bus [4, 13, 21].

A study by [22] followed a similar methodology, observing that most studies concentrate on a model in which students are transported to a single school. However, the problem becomes more complicated and can be viewed as a VRP with mixed load when there are multiple schools involved in delivering students. Students may take the same bus to and from their respective locations, allowing for more efficient use of transportation resources.

Further studies have been carried out which connect the single or multiple school problem to bus stop selection and route generation by [4, 21]. Assigning students to particular bus stops and planning appropriate routes is the primary concern in the single-school problem. Students should be kept from being picked up or put off in the correct location, and the bus's carrying capacity should be maintained. In the multiple school problem, children from different schools are combined into one bus load, necessitating the optimisation of pick-up and drop-off locations.

B. Service Surrounding (Urban or Rural)

The area of urban, rural, or a hybrid of both has a substantial impact on the type of problems that are experienced [4]. Many students in urban areas live within a comfortable walking distance of a central bus stop because of the greater concentration of students. It is also observed by a study from [23] that various challenges arise when it comes to providing transportation for students in both urban and rural areas. In rural areas, students often have to contend with steep, dirty, and narrow roads, which means that smaller vehicles are typically used to transport them. Besides considering a heterogeneous fleet while modelling a rural school bus routing problem, different speeds for different types of vehicles, mixed load, and multi-loading must be accounted for to provide greater flexibility.

In a rural scenario, each bus will initially stop at several stations to pick up students from the same region before continuing to their respective schools to unload them. In contrast, the exact location of the bus stops in an urban area is initially determined by considering the locations of the students who make up the majority of the student population. According to the research conducted by [24], the selection of bus stops is connected to the service environment. It is expected that the density of student housing will be significantly higher in urban areas than it will be in rural areas. This indicates that there will probably be a greater requirement to construct several school stations in urban areas to serve the population. On the other hand, limitations imposed in urban areas are less severe in rural areas because lower student population. Thus, they can be picked up directly from their houses by school buses, which can significantly reduce the number of required school stations.

C. Mixed-load

[24, 25, 26] investigated this problem to reduce the average amount of time students spend taking the bus and decrease the number of active buses. To find a solution to this problem, [24] examined a hybrid multi-objective ACO (Ant Colony Optimisation) and a unique routing heuristic algorithm to minimise the amount of time spent on the task. The multi-loading school bus routing problem was presented by [26]. This problem extends the rural bus routing problem with mixed loads to make it possible for students from multiple schools to use the same bus simultaneously, regardless of which shift or direction they are travelling.

Moreover, [25] emphasised that the mixed-loading assumption increases flexibility while simultaneously contributing to a decrease in total cost. They developed a solution approach to managing the problem of mixed loading and evaluated how effective it was based on the number of needed vehicles. They used the proposed algorithm to solve a few real-world situations, and the results showed a reduction in the total number of vehicles compared to the experimental plans. While [27] concentrates the majority of their attention on the single-load variation of the school bus routing problem, which is a more restricted alternative to the mixed-load option. This variant makes the assumption that students from different schools are unable to travel together on the same bus, which leads to greater costs from an operational point of view. To address and ultimately solve the problem of single-load school bus routing, they created a two-step heuristic method that considers trip compatibility to solve various vehicle routing problems.

D. Fleet Mix (HO & HT)

The Homogeneous Fleet (HO) refers to a group of buses that are identical or similar in terms of their design, performance, and specifications. In this research, the characteristics include costs, capacity, maximum travelling time, travelling distance, etc. The benefits of a Homogeneous Fleet are to focus on reducing the cost of bus operation and to improve fleet management. Additionally, the researchers found out that 14 authors chose Homogeneous Fleet in their modelling, and they separated in three different ways: some of them were focused on reducing cost [9, 10, 11, 14, 26, 28, 29, 30]. The others were focused on reduced total numbers of used buses [2, 3, 7, 9, 11, 15, 19, 27]. The rest focused on the shortest travel distance (STD) [2, 11, 28, 31]. The Heterogeneous fleet (HT) refers to all the characteristics in the opposite way of HO, but they do have a similarity of the final destination to the school. [4] in the case of the same bus capacity, HO is easier to meet the time requirement than HT fleets. HT fleets require more buses to cover the same number of passengers as HO fleets, but they may also provide more flexibility in terms of scheduling and stop locations.

To sum up, both HT and HO fleets can be selected for the solution of SBRP. For example, the HT fleets can be selected if the configuration is to transport all or the majority type of students or to multiple schools. Homogeneous loads can be chosen to serve students with only a single school or limited route planning. The type of fleets with characteristics may affect solutions for bus route generation and bus route schedule.

E. Objectives

Min D	To minimise the total distance spent
Min N	To minimise the total number of buses used
Min T	To minimise the total time spent
Min C	To minimise the total cost

Recent publications that focus on the sub-problem of generating bus routes have mostly aimed to enhance the efficiency of routing plans by reducing costs. These publications typically propose a model to minimise one particular aspect of the overall cost in Table II, 10 refers to minimise the total cost of bus operation (Min C), 17 aimed to minimise the total number of buses used (Min N). It should be noted that some publications offer multiple models, which include two or more objectives instead of just one. [32] introduced a new approach to address the School Bus Scheduling Problem (SBSP), where the goal is to minimise transportation costs by optimising the school start times and bus operation times. 19 publications focus on minimising the total distance spent that a bus route operation between two stops. [33] focused on minimising total bus travel distance (Min D) while satisfying capacity and load constraints. [7] mentioned minimising the total number of buses and the total travel time/distance to solve the SBRP. And 14 publications are aimed at minimising the total time spent in their study, they believe it helps to optimize the equation from SBRP modelling.

Table II: Objectives and constraints.

References	Objectives	Constraints
Jaradat and Shatnawi (2020)	Min T, Min C, Min D	BC, TW, DOR
Oluwadare, Oguntuyi and Nwaiwu (2018)	Min N, Min D	BC, TW
Ellegood *et al.* (2020)	Min D, Min T, Min C	BC, MRT, TW, MWTD, EPT, MSN, BS
Noor *et al.* (2020)	Min D	WT, C
Ali Shafahi *et al.* (2018a)	Min N, Min T, Min D	MRT, BC, TW
Armond *et al.* (2022)	Min D	N, DOR, BC
Han and Zhang (2019)	Min N	BC, DOR
Guo *et al.* (2019)	Min C	BC, TW, C
Ozmen and Sahin (2021)	Min D, Min N	BC, C
Banerjee and Smilowitz (2019)	Min N, Min T	N
Sciortino (2022)	Min N, Min T, Min D	BC, MWTD
Bertsimas *et al.* (2019)	Min D, Min T, Min N	MWTD
Ochoa-Zezzatti *et al.* (2020)	Min D, Min T, Min N	BS, WT
Ren *et al.* (2019)	Min T, Min N	MWTD, SWD, BC, MRT
Li and Chow (2021)	TSD	C, WT, NBR
Shang *et al.* (2021)	Min N, Min T, Min D	BC, TW
Ansari *et al.* (2021)	Min T, Min D	EPT, BC
Miranda *et al.* (2021)	Min N, Min T, Min C	TW, MWTD, BC, C
Mokhtari and Ghezavati (2018)	Min N, Min T	MRT, TW
Komijan *et al.* (2021)	Min N, Min D, Min T	BC, MRT, TW, MWTD, EPT
Miranda *et al.* (2018)	Min C	TW, BC, MRT, MWTD
Ali Shafahi *et al.* (2018b)	Min N, Min T, Min D	MRT, BC, TW
Ümit *et al.* (2019)	Min C	BC, SWD
Prah *et al.* (2018)	Min C	SWD
Sánchez-Ansola *et al.* (2022)	Min D	BS, Min D
Calvete *et al.* (2020)	Min C	Min C, Min T, Min D, Min N
Lewis *et al.* (2018)	Min N	BC, WT
Hou *et al.* (2022)	Min N, Min D	TW, DOR, EPT
Guo *et al.* (2022)	Min C, Min N	BC, SWD
Dang *et al.* (2019)	Min N, Min D	BC, Min T
Calvete *et al.* (2022)	Min C, Min D	BC, Min N
Hou *et al.* (2022)	Min D, Min N	Min C, Min N, TW, DOR, EPT
Sales *et al.* (2018)	Min C	BC, SWD

Moreover, from a total of 18 journals related to Min D, only 3 of them was used for multiple schools [12, 20, 27], and 9 of them for a single school.

From the referenced journals, half of the journals (10 of 33) have more than 3 objectives and mostly, they are mentioned with minimised time (Min T), minimised number of buses used (Min N), minimised distance (Min D), and most importantly, the minimised total cost (Min C). [21] improved the optimization of student-bus assignments and bus routing, an extended-state dimension has been introduced to account for the number of students who are travelling to different schools via buses. This allows for more efficient and effective routing decisions to be made, which can lead to cost savings and better service for students. [12] with an objective to minimise the total distance (Min D), noted that traditional approaches to bus schedules are often inefficient and time-consuming and proposed their new method as a more effective alternative, it involves using a reinforcement learning algorithm to learn from previous bus schedules and make adjustments in real-time. [33] mentioned that time is important that traditional methods of dispatching buses often result in running late or early, which can lead to inconvenience and frustration for passengers and they provided a mathematical optimization algorithm to determine the best dispatching times for buses based on a number of factors, including passenger demand, travel times, and bus capacity. The objective of minimising the distance (Min D) was the most mentioned element from the research, more than half of the publications (19 out of 33) chose to minimise the total distance as their goal in their study.

F. Constraints

BC	The Capacity of a bus
MRT	The Maximum Riding Time
TW	Time windows between each bus arrival at each stop
MWTD	The maximum walking time distance between two stops
EPT	The Earliest Pick-up Time of a Bus
MSN	The Minimum Student Number to Create a Route
DOR	The Distance of the Route
ATES	The Average Time for Each Stop
WT	The Waiting Time
SWD	The Student Walking Distance
BS	The Number of Bus Stops
N	The Number of Bus Used
C	Cost

Four constraints are the most mentioned in this study, like bus capacity (BC), time windows (TW), maximum ride time (MRT), and maximum walking time or distance (MWTD). The constraints of capacity were mentioned 21 times, and the time window constraint is typically used to ensure that students are picked up and dropped off within a specific time frame. The Maximum Ride Time (MRT) constraint specifies the maximum time that a student can spend on a school bus during a single trip. The time window (TW) and the earliest pick-up time of a bus (EPT) constraints are usually defined to ensure that students do not spend too much time waiting for the bus, and the window will limit the arrival time between each stop for a bus route. And those publications that include these two constraints, will also focus on the average time for each stop (ATES) and the number of bus stops (BS). It will help to develop an efficiency-optimizing SBRP formula in their study and the final result will be accurate with these constraints included.

[12] developed an ITS that can generate high-quality schedules while reducing the total distance travelled and the number of buses used, it also can help school transportation planners to develop more efficient and effective school bus schedules, thereby reducing transportation costs, improving student safety, and enhancing the overall quality of school transportation services. The purpose of MRT is to minimise the total travel time for students while also considering the need to limit the duration of their bus rides. From Table 2, the MRT is only used in 7 out of 34 publications [4, 7, 19, 24, 25, 26, 27]. In the investigation conducted by [20], the mixed ride approach tends to return solutions with fewer vehicles and fewer bus stops, less average travel distance, and shorter average travel time. In the newest studies, some constraints have been mentioned with new elements related to SBRP articles: maximum walking time or distance (MWTD), earliest pick-up time (EPT), and minimum student number to create a route (MSN). The distance of the route (DOR) is a key element that can help in developing an efficiency model for SBRP, 5 publications [1, 6, 7, 29, 34] mentioned this in their study. With the selection of HO fleets, the constraints of MWTD were only mentioned in two publications for a single school [29, 35]. Study [28] reveals that the share-ability network-based approach is effective in reducing pickup time (EPT) and improving the solution quality and it can handle large-scale datasets with up to 10,000 students and 300 buses.

The number of buses used (N) can be used in both objectives and constraints for generating the bus schedule and bus routing. [13] found out that when equity constraints that every student can use the bus service were included in the scheduling model, the total transportation cost increased slightly, but the resulting schedules were more equitable and accessible for all students.

G. Example Diagram of Mixed-load Scenario (Illustrated for this study)

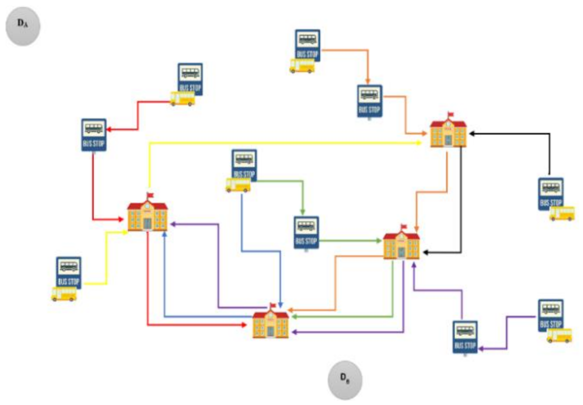

Fig. 2. Mixed-load scenario.

In the Fig. 2, there are 4 schools represented School 1, School 2, School 3, and School 4. Additionally, there are 10 bus stops marked as Bus Stop 1, Bus Stop 2, and so on up to Bus Stop 10. There are a total of 7 buses denoted as Bus 1, Bus 2, and so on up to Bus 7. 2 depots are indicated as Depot A and Depot B. The depot refers to a facility or location where vehicles, equipment, or goods are stored, maintained, or distributed.

The SBRP involves determining the most efficient routes for school buses to transport students from the various schools to their designated bus stops. Each bus is assigned to a specific route, and the buses are connected to the corresponding bus stops based on the route assignments. For example, Bus 1 is connected to Bus Stop 2 through Route 1, indicating that it serves as the transportation vehicle for students travelling from Bus Stop 1 to School 1.

IV. APPLICATION OF MATHEMATICAL MODEL

A. GAMS Software

[36, 37, 38] indicated that GAMS (General Algebraic Modelling) is advanced mathematical software that is simple to put into practice for optimization and modelling problems comprising non-linear, linear and mixed integer programming models. One well-known study that is cited often in research on GAMS is that of [39], who found GAMS has been developed for solving complex Mixed Complementarity Problems (MCP) as it is also one of the mathematical optimization models. GAMS is capable of overcoming mathematical problems that are both large and intricate [37]. This flexible modelling language makes use of a concise approach to manage complex models and reduce the least possible on occurring errors in the process of generating the solution. Applying GAMS in a real-case situation and investigating results or using more efficient methods (e.g., meta-heuristics) and comparing the efficiency of their results with some statistical analyses are recommended for future studies [18].

B. The mathematical Model Classification

This paper mainly studies how a mathematical model can be used to support the school bus routing problem (SBRP). The classification of the mathematical model comprises exact, heuristic and metaheuristic algorithms. Discussing each of the related examples of mathematical modelling problems and methods as tabulated in Table III.

Table III: Solution methods.

References	**Solution Methods**	
	Exact	***Heuristic***
Jaradat and Shatnawi (2020)		√
Ellegood *et al.* (2020)		√
Noor *et al.* (2020)		√
Armond *et al.* (2022)		√
Ozmen and Sahin (2021)		√
Sciortino (2022)		√
Ochoa-Zezzatti *et al.* (2020)		√
Li and Chow (2021)		√
Shang *et al.* (2021)		√
Ansari *et al.* (2021)		√
Miranda *et al.* (2021)		√
Komijan *et al.* (2021)	√	√
Sánchez-Ansola *et al.* (2022)		√
Calvete *et al.* (2020)		√
Hou *et al.* (2022)		√
Guo *et al.* (2022)	√	
Calvete *et al.* (2022)		√
Hou *et al.* (2022)		√

1) Exact algorithm

[31] presented the final outcome of the exact algorithm will be the exact solution but not an optimal solution. The exact algorithm in this review will discuss branch-and-bound and cutting-plane methods. The branch-and-cut algorithm is a fusion of branch-and-bound and cutting-plane techniques that are used to solve a sequence of relaxation problems in Integer Linear Programming (ILP) [40, 41], as cited in [10]. The branch and bound method is an exact algorithm often used to solve the problem of IP and MIP. Based on the analysis in [42] proposed integer programming formulation for an SBRP that involves the bus stop selection and students' allocation to stops which will further research focusing on a metaheuristic optimization method and cutting plane algorithm to add new features like time window constraints as well as more buses are allowed to reach to a bus stop. SBRP has applied an integer programming method for achieving the aims of students' transportation cost minimization in light of factors like distance and capacity [43, 44] as cited in [3].

2) Heuristic algorithm

[31] claimed heuristic algorithms are unable to accurately calculate the exact value but it can provide a feasible solution by fitting the actual situation. [45] presented a hybrid algorithm combining the limitations and strengths of both the Bat algorithm (BA) and adaptive particle swarm algorithm to solve the public bus route problem in rural areas. A mixed integer programming model is set up by a heuristics algorithm in order to analyse the shortest distribution path and time window [31]. [46] introduces the gravity model, a heuristic algorithm to optimise typical NP-hard bus route scheduling problems by generating the final route using intra-route as well as inter-route optimization algorithms.

3) Meta-heuristic algorithm

[26] as cited in [22] claimed that locally search-based meta-heuristic ways are used in large-scale factual cases to solve the multi-loading SBRP. The Tabu search (TS) algorithm was first proposed by [47] as cited in [10] presenting that TS has emerged as a potent metaheuristic which can effectively tackle combinatorial optimization problems by extending the local search algorithm, as it will avoid the previously explored paths. [48] solved the problem of vehicle routing with a time window by implementing two-phase hybrid meta-heuristics. Although shortening the total distance and decreasing the car number by applying a metaheuristic in control of a neighbourhood search, it has limitations on this scope once a single route has to take a large number of passengers on board [48]. In the studies of [2, 30], as cited in [10] presented that a genetic algorithm (GA) is one of the metaheuristic methods that apply in SBRP imitates the genetics of living things to implement the survival of the fittest principle to choose the best fittest route solutions called chromosomes. [30] generates the optimal solution by applying the combination of GA and Simulated Annealing Algorithm (SAA) meanwhile taking some constraints including actual time departure, arrival time and limited capacity into consideration.

V. CONCLUSION

In conclusion, the school must determine the optimum routes for students to be picked up and dropped off, and the School Bus Routing Problem is a task that involves finding the optimal routes. However, this is a challenging task because many factors need to be considered, such as traffic, bus capacity, and the location of bus stops. Many researchers are working together to find new ways to solve the SBRP, primarily through exact, heuristic, and metaheuristic algorithms, which can find better solutions quickly. Moreover, it is also important to note that choosing an appropriate bus stop location is another essential part of the SBRP for picking up students.

The SBRP's sub-problems can be divided into four categories: the number of schools, service surroundings, mixed-load, and fleet mix. To address these issues, the authors have set up multiple objectives for solving SBRP. To address these issues, the authors have set up multiple objectives for solving SBRP. The survey findings show that the most common limitations were:

- Maximum riding time and distances.
- Maximum bus capacities.
- Time windows.
- Walking times or distances.

Thus, the researchers have been exploring new solutions for these challenges. Hence, metaheuristic solution methods are becoming increasingly popular as a way of addressing more complicated SBRP problems.

APPENDIX

Categories	Characteristics	Abbreviation
Sub-problem Types	Bus Route Generation	BRG
	Bus Route Scheduling	BRS
	Bus Stop Selection	BSS
Number of Schools	Single School	S
	Multiple School	M
Environment	Urban	U
	Rural	R
Fleet Mix	Homogenous	HO
	Heterogenous	HT
Objectives	Minimise Number of Bus Used	Min N
	Minimise Travel Distance	Min D
	Minimise Travel Time	Min T
	Minimise Cost	Min C
Constraints	Bus Capacity	BC
	Maximum Riding Time	MRT
	Time Window	TW
	Minimise walking time or distance	MWTD
	Earliest Pick-up Time	EPT
	Minimum Student Number to Create a Route	MSN
	Distance of the Route	DOR
	Average Time for Each Stop	ATES
	Waiting Time	WT
	Student Walking Distance	SWD
	Number of Bus Stops	BS
	Number of Bus Used	N
	Cost	C

REFERENCES

[1] R. M. Newton and W. H. Thomas, “Design of School Bus Routes by Computer,” *Socio-Economic Planning Sci.*, vol. 3, no. 1, pp. 75–85, 1969.

[2] S. A. Oluwadare, I. P. Oguntuyi and J. C. Nwaiwu, “Solving School Bus Routing Problem using Genetic Algorithm-based Model,” *Int. J. Intellig. Sys. and Appl.*, vol. 10, no. 3, pp. 50–58, 2018.

[3] A. S. Jaradat and M. Q. Shatnawi, “Solving School Bus Routing Problem by Intelligent Water Drops Algorithm,” *J. Comput. Sci.*, vol. 16, no. 1, pp. 25–34, 2020.

[4] R. M. Noor, N. B. G. Rasyidi, T. Nandy and R. Kolandaisamy, “Campus Shuttle Bus Route Optimization Using Machine Learning Predictive Analysis: A Case Study,” *Sustainability*, vol. 13, no. 1, pp. 225, 2020.

[5] W. A. Ellegood, S. Solomon, J. North and J. F. Campbell, “School Bus Routing Problem: Contemporary Trends and Research Directions,” *Omega*, vol. 95, pp. 102056, 2020.

[6] J. Tang, Y. Yang and Y. Qi, “A Hybrid Algorithm for Urban Transit Schedule Optimization,” *Physica A: Statistic. Mechanic. and its Appl.*, vol. 512, pp. 745–755, 2018.

[7] A. Shafahi, Z. Wang and A. Haghani, “SpeedRoute: Fast, Efficient Solutions for School Bus Routing Problems,” *Transport. Res. Part B: Methodological*, vol. 117, pp. 473–493, 2018.

[8] A. M. Armond, Y. D. Prasetyo and W. Ediningrum, “Application of Ant Colony Optimization (ACO) Algorithm to Optimize Trans Banyumas Bus Routes,” in *2022 IEEE Int. Conf. Cybernetics and Comput. Intellig.*, Malang, Indonesia, pp. 132–137, 2022.

[9] X. Han and X. Zhang, “School Bus Route Optimization Based on Improved Ant Colony Algorithm,” in *2019 4th Int. Conf. Electromechanic. Contr. Technol. and Transport.*, Guilin, China, pp. 312–316, 2019.

[10] R. Guo, W. Guan, W. Zhang, F. Meng and Z. Zhang, “Customized Bus Routing Problem with Time Window Restrictions: Model and Case Study,” *Transportmetrica A: Transport Sci.*, vol. 15, no. 2, pp. 1804–1824, 2019.

[11] M. Ozmen and H. Sahin, “Real-Time Optimization of School Bus Routing Problem in Smart Cities Using Genetic Algorithm,” in *2021 6th Int. Conf. Inventive Comput. Technol.*, Coimbatore, India, pp. 1152–1158, 2021.

[12] E. Köksal Ahmed, Z. Li, B. Veeravalli and S. Ren, “Reinforcement Learning-Enabled Genetic Algorithm for School Bus Scheduling,” *J. Intellig. Transport. Sys.*, vol. 26, no. 3, pp. 269–283, 2022.

[13] D. Banerjee and K. Smilowitz, “Incorporating Equity into the School Bus Scheduling Problem,” *Transport. Res. Part E: Logistics and Transport. Rev.*, vol. 131, no. 3, pp. 228-246, 2019.

[14] M. Babaei and M. Rajabi-Bahaabadi, “School Bus Routing and Scheduling with Stochastic Time-Dependent Travel Times Considering On-Time Arrival Reliability*,” Comput. & Indust, Eng.*, vol. 138, pp. 106125, 2019.

[15] M. Sciortino, R. Lewis and J. Thompson, “A School Bus Routing Heuristic Algorithm Allowing Heterogeneous Fleets and Bus Stop Selection,” *SN Comput. Sci.*, vol. 4, no. 1, pp. 74, 2022.

[16] D. Bertsimas, A. Delarue and S. Martin, “Optimizing Schools’ Start Time and Bus Routes,” *Proc. Nat. Acad. Sci.*, vol. 116, no. 13, pp. 5943–5948, 2019.

[17] A. Ochoa-Zezzatti, U. Carbajal, O. Castillo, J. Mejía, G. Rivera and S. Gonzalez, “Development of A Java Library to Solve the School Bus Routing Problem,” *Smart Technol. Smart Cities*, pp. 175–196, 2020.

[18] M. R. Sayyari, R. Tavakkoli-Moghaddam, A. Abraham and N. Oladzad-Abbasabady, “A School Bus Routing and Scheduling Problem with Time Windows and Possibility of Outsourcing with the Provided Service Quality,” *Intellig. Sys. Design and Appl.*, vol. 1351, pp. 829–839, 2021.

[19] J. Ren, W. Jin and W. Wu, “A Two-Stage Algorithm for School Bus Stop Location and Routing Problem with Walking Accessibility and Mixed Load,” *IEEE Access*, vol. 7, pp. 119519–119540, 2019.

[20] M. Li and J. Y. J. Chow, “School Bus Routing Problem with A Mixed Ride, Mixed Load, and Heterogeneous Fleet,” *Transport. Res. Rec.*, vol. 2675, no. 7, pp. 467–479, 2021.

[21] P. Shang, L. Yang, Z. Zeng and L. (Carol) Tong, “Solving School Bus Routing Problem with Mixed-Load Allowance for Multiple Schools,” *Comput. & Indust. Eng.*, vol. 151, pp. 106916, 2021.

[22] A. Ansari, L. Farrokhvar and B. Kamali, “Integrated Student to School Assignment and School Bus Routing Problem for Special Needs Students,” *Transport. Res. Part E: Logistics and Transport. Rev.*, vol. 152, pp. 102416, 2021.

[23] D. M. Miranda, R. S. de Camargo, S. V. Conceição, M. F. Porto and N. T. R. Nunes, “A Metaheuristic for The Rural School Bus Routing Problem with Bell Adjustment,” *Expert Sys. with Appl.*, vol. 180, pp. 115086, 2021.

[24] N. Mokhtari and V. Ghezavati, “Integration of Efficient Multi-Objective Ant-Colony and A Heuristic Method to Solve A Novel Multi-Objective Mixed Load School Bus Routing Model,” *Appl. Soft Comput.*, vol. 68, pp. 92–109, 2018.

[25] A. Rashidi Komijan, P. Ghasemi, K. Khalili-Damghani and F. HashemiYazdi, “A New School Bus Routing Problem Considering Gender Separation, Special Students and Mix Loading: A Genetic Algorithm Approach,” *J. Optimiz. Indust. Eng.*, vol. 14, no. 2, pp. 23–39, 2021.

[26] D. M. Miranda, R. S. de Camargo, S. V. Conceição, M. F. Porto and N. T. R. Nunes, “A Multi-Loading School Bus Routing Problem,” *Expert Sys. with Appl.*, vol. 101, pp. 228–242, 2018.

[27] A. Shafahi, Z. Wang and A. Haghani, “A Matching-based Heuristic Algorithm for School Bus Routing Problems,” *ArXiv*, vol. abs/1807.05311, 2018.

[28] X. Guo and S. Samaranayake, “Shareability Network Based Decomposition Approach for Solving Large-Scale Single

School Routing Problems," *Transport. Res. Part C: Emerging Technol.*, vol. 140, pp. 103691, 2022.

[29] L. Dang. Y. Hou and Q. Liu, "A Hybrid Metaheuristic Algorithm for the Bi-objective School Bus Routing Problem," [Available Online: 12 Aug 2019] https://www.semanticscholar.org/paper/A-Hybrid-Metaheuristic-Algorithm-for-the-School-Bus-Dang-Hou/7b8e12db4d601fb8c8169967b0078bafb7df5b2b.

[30] S. Thangiah, "Vehicle Routing with Time Windows using Genetic Algorithms," *The Practical Handbook of Genetic Algorithms*, L. Chambers, Ed., CRC Press, 1995.

[31] F. Wu and Y. Zhu, "Transportation Route Optimization Based on Heuristic Algorithm," *J. Phys.: Conf. Ser.*, vol. 2083, no. 3, pp. 032016, 2021.

[32] L. Zeng, S. Chopra and K. Smilowitz, "A Bounded Formulation for The School Bus Scheduling Problem," *Transport. Sci.*, vol. 56, no. 5, pp. 1148–1164, 2022.

[33] Ü. G. Ümit and F. Kılıç, "A School Bus Routing Problem Using Genetic Algorithm by Reducing the Number of Buses," in *2019 Innov. Intellig. Sys. and Appl. Conf.*, Izmir, Turkey, pp. 1-6, 2019.

[34] H. I. Calvete, C. Galé, J. A. Iranzo and P. Toth, "A Partial Allocation Local Search Matheuristic for Solving the School Bus Routing Problem with Bus Stop Selection," *Mathematics*, vol. 8, no. 8, pp. 1214, 2020.

[35] H. I. Calvete, C. Galé and J. A. Iranzo, "Approaching The Pareto Front in A Biobjective Bus Route Design Problem Dealing with Routing Cost and Individuals' Walking Distance by using A Novel Evolutionary Algorithm," *Mathematics*, vol. 10, no. 9, pp. 1390, 2022.

[36] D. Chattopadhyay, "Application of General Algebraic Modeling System to Power System Optimization," *IEEE Trans. Power Sys.*, vol. 14, no. 1, pp. 15–22, 1999.

[37] D. Bisen and H. M. Dubey, "Dynamic Economic Load Dispatch with Emission and Loss using GAMS," *Int. J. Eng. Res. and Technol.*, vol. 1, IJERTV1IS3127, 2012.

[38] E. Salari and A. Askarzadeh, "A New Solution for Loading Optimization of Multi-Chiller Systems by General Algebraic Modeling System," *Appl. Thermal Eng.*, vol. 84, pp. 429–436, 2015.

[39] M. R. Bussieck and A. Meeraus, "General Algebraic Modeling System (GAMS)," in *Modeling Languages in Mathematical Optimization*, J. Kallrath, Ed., Boston, Springer US, pp. 137–157, 2004.

[40] M. Fischetti, J. J. Salazar González and P. Toth, "A Branch-and-Cut Algorithm for the Symmetric Generalized Traveling Salesman Problem," *Operation. Res.*, vol. 45, no. 3, pp. 378–394, 1997.

[41] J. Lysgaard, A. N. Letchford and R. W. Eglese, "A New Branch-And-Cut Algorithm for The Capacitated Vehicle Routing Problem," *Mathematic. Program.*, vol. 100, no. 2, pp. 423–445, 2004.

[42] P. Schittekat, M. Sevaux and K. Sorensen, "A Mathematical Formulation for A School Bus Routing Problem," in *2006 Int. Conf. Serv. Sys. and Serv. Manage.*, Troyes, France, pp. 1552–1557, 2006.

[43] T. Bektaş and S. Elmastaş, "Solving School Bus Routing Problems Through Integer Programming," *J. Operation. Res. Soc.*, vol. 58, no. 12, pp. 1599–1604, 2007.

[44] A. Fügenschuh, "Solving A School Bus Scheduling Problem with Integer Programming," *European J. Operation. Res.*, vol. 193, no. 3, pp. 867–884, 2009.

[45] B. Zhang, Z. Zhong, X. Zhou, Y. Qu and F. Li, "Optimization Model and Solution Algorithm for Rural Customized Bus Route Operation under Multiple Constraints," *Sustainability*, vol. 15, no. 5, pp. 3883, 2023.

[46] J. Sun, Y. Chen, J. Huang, P. Wei and C. Song, "Flexible Bus Route Optimization Scheduling Model," *Advances in Civil Engineering*, vol. 2021, pp. 1–13, 2021.

[47] F. Glover, "Tabu Search—Part I," *ORSA J. Comput.*, vol. 1, no. 3, pp. 190–206, 1989.

[48] J. Homberger and H. Gehring, "A Two-Phase Hybrid Metaheuristic for the Vehicle Routing Problem with Time Windows," *European J. Operation. Res.*, vol. 162, no. 1, pp. 220–238, 2005.