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Neighbourly Edge Irregularity on Interval-valued Pythagorean Neutrosophic Graph

Murugappan Mullai*, Govindan Vetrivel, Grienggrai Rajchakit* and G. Madhan Kumar

***Abstract* – Interval-Valued Pythagorean Neutrosophic Graph (IVPNG) comprises independent and dependent membership elements of interval entries. This study aims to extend the findings of the fuzzy concept to a neutrosophic environment, which is helpful in examining a problem or an event with three associated memberships. This paper demonstrates some kinds of irregular properties on IVPNG, like neighbourly irregular, neighbourly totally irregular, strongly irregular, strongly totally irregular, highly irregular, and highly totally irregular. On certain conditions, neighbourly edge irregular and neighbourly edge totally irregular IVPNG satisfy these irregular properties and vice versa.**

Keywords—Neutrosophic graph, Interval-valued pythagorean neutrosophic graph, Edge irregular IVPNG, Neighbourly edge irregular IVPNG, Neighbourly edge totally irregular IVPNG.

I. INTRODUCTION

L.A. Zadeh's contribution [14] of fuzzy set and fuzzy relation led to the foundation of Rosenfeld's fuzzy graph and its applications. A. NagoorGani and K. Radha [9] elaborated the properties of fuzzy work by introducing the regularity of fuzzy graphs, which considers the equivalent value in degree and totally degree. The irregularity concept on fuzzy graphs was initiated and extended to find neighbourly irregular and highly irregular fuzzy graphs by A. NagoorGani and S.R. Latha [10]. Strongly irregular properties on fuzzy graphs and their properties were illustrated in detail by S.P. Nandhini and E. Nandhini [11]. The regularity of edge degree and total edge degree in fuzzy graphs and their extension to interval-valued fuzzy graph (IVFG) was presented by K. Radha and N. Kumaravel. S. Ravi Narayanan and N.R. Santhi Maheswari extended the theory by analyzing the irregularity property on edge, edge totally, strongly edge, and strongly edge totally IVFG. With these sources, B. Sundarapandian and S. Ravi Narayanan developed the neighbourly edge irregular and neighborly edge totally irregular IVFG.

K.T. Atanassov explored the theory of intuitionistic fuzzy set from a fuzzy set by adding a non-membership function and defined an intuitionistic fuzzy graph. Later, Atanassov and Gargov jointly developed the interval-valued intuitionistic fuzzy set, which was refined into defining the interval-valued intuitionistic fuzzy graph.

Smarandache [12,13] came up with an additional membership function called "indeterminacy" and defined the neutrosophic set, and later, it emerged as a neutrosophic graph. S. Broumi and Smarandache have worked jointly on Single valued neutrosophic graph and its properties. S. Broumi et al. [5,6] coined the term "Interval-valued neutrosophic graph," which is the generalization of the above all mentioned graph types. D. Ajay and P. Chellamani [1] restricted the independent memberships of a neutrosophic graph by introducing a

*Corresponding author. Email: mullaim@alagappauniversity.ac.in, kreangkri@mju.ac.th

Murugappan Mullai, Assistant Professor (DDE), Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.

Govindan Vetrivel, Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.

Grienggrai Rajchakit, Associate Professor, Department of Mathematics, Faculty of Science, Maejo University, Chiang Mai 50290, Thailand.

G. Madhan Kumar, Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.

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Pythagorean neutrosophic fuzzy graph, which bears the dependent component.

This paper deals with the initiation of neighbourly edge irregular interval-valued Pythagorean neutrosophic graphs (NEI-IVPNG), neighbourly edge totally irregular interval-valued Pythagorean neutrosophic graphs (NETI-IVPNG), and finally, a comparative study between NEI-IVPNG and NETI-IVPNG is done and also analyzed with SI-IVPNG, HI-IVPNG.

II. PRELIMINARIES

Definition 2.1.[1]

A Pythagorean Neutrosophic Graph G is defined to be a set G = (A, B), where
(i) The functions M_A : V → [0,1], I_A : V → [0,1] and N_A : V → [0,1] represent the degree of membership, indeterminacy membership and non-membership of the element x ∈ V, respectively, such that 0 ≤ $M_A(\mathrm{x})^2$+ $I_A(\mathrm{x})^2$+ $N_A(\mathrm{x})^2$ ≤ 2, for all x ∈ V ,
(ii) The functions M_B : E ⊆ V×V → [0,1], I_B : E ⊆ V×V → [0,1] and N_B : E ⊆ V×V → [0,1] are defined by
M_B(xy) ≤ min{ M_A(x), M_A(y)},
I_B(xy) ≤ min{I_A(x), I_A(y)} and
N_B(xy) ≥ max{N_A(x), N_A (y)} such that
0 ≤ $M_B(\mathrm{xy})^2$+ $I_B(\mathrm{xy})^2$+ $N_B(\mathrm{xy})^2$ ≤ 2, for all xy ∈ E

Definition 2.2.

Let G = (A, B) be an Interval-Valued Pythagorean Neutrosophic Graph (IVPNG), where
(i) The functions M_A : V → D[0,1], I_A : V → D[0,1] and N_A : V → D[0,1] represent the degree of interval-valued membership,interval-valued indeterminacy membership and interval-valued non-membership of the element x ∈ V,respectively,such that $M_{A+}(\mathrm{x})^2$+ $I_{A+}(\mathrm{x})^2$+ $N_{A+}(\mathrm{x})^2$ ≤ 2 , $M_{A-}(\mathrm{x})^2$+ $I_{A-}(\mathrm{x})^2$+ $N_{A-}(\mathrm{x})^2$ ≤ 2 for all x ∈V,
(ii) The functions M_B : E ⊆ V×V → D[0,1], I_B : E ⊆ V×V → D[0,1] and N_B : E ⊆ V×V → D[0,1] are defined by
M_{B-}(xy)≤ min{M_{A-}(x), M_{A-}(y)};
M_{B+}(xy)≤ min{M_{A+}(x), M_{A+}(y)}
I_{B-}(xy)≤ min{I_{A-}(x), I_{A-}(y)};
I_{B+}(xy)≤ min{I_{A+}(x), I_{A+}(y)}
N_{B-}(xy)≥ max{N_{A-}(x), N_{A-}(y)};
N_{B+}(xy)≥ max{N_{A+}(x), N_{A+}(y)} such that
$M_{B+}(\mathrm{xy})^2$+ $I_{B+}(\mathrm{xy})^2$+ $N_{B+}(\mathrm{xy})^2$ ≤ 2, $M_{B-}(\mathrm{xy})^2$+ $I_{B-}(\mathrm{xy})^2$+ $N_{B-}(\mathrm{xy})^2$ ≤ 2 for all xy ∈ E. Here, A is the interval-valued pythagorean neutrosophic vertex set of G and B is the interval-valued pythagorean neutrosophic edge set of G.

Defintion 2.3.

If all set of adjacent edges of a connected IVPNG have distinct degree then the graph is known as NEI-IVPNG.

Defintion 2.4.

If all set of adjacent edges of a connected IVPNG have distinct total degree then the graph is known as NETI-IVPNG.

Example 2.5.

Let G: (V, E) be an IVPNG. From Figure 1, d(u)= d(v)= d(w)= d(x)= ⟨ (0.3,0.5), (0.7,0.7), (0.5,1) ⟩, d(uv) = d(wx) = ⟨ (0.4,0.6), (0.8,0.8), (0.4,0.8) ⟩ and d(vw) = d(xu) = ⟨ (0.2,0.4), (0.6,0.6), (0.6,1.2) ⟩.
It shows that, all set of adjacent edges have distinct degree.
Therefore, G is concluded as an NEI-IVPNG.
Total degrees of the edges are listed as follows:
td(uv) = td(wx) = ⟨ (0.5,0.8), (1.1,1.1), (0.7,1.4) ⟩,
td(vw) = td(xu) = ⟨ (0.4,0.7), (1.0,1.0), (0.8,1.6) ⟩.
It shows that, all set of adjacent edges have distinct total degree. Therefore, G is a NETI-IVPNG.
Hence, the graph G is both NEI-IVPNG and NETI-IVPNG.
The following graph is an example for both NEI-IVPNG and NETI-IVPN:

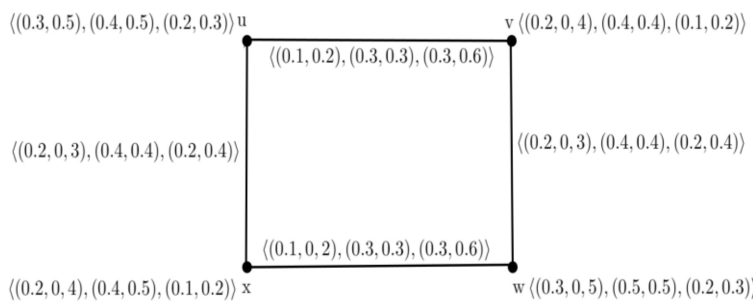

Fig.1.

Defintion 2.6.

Let G = (A, B) denotes a connected IVPNG. Then G is said to be a strongly irregular interval valued pythagorean neutrosophic graph (SI-IVPNG) if all set of vertices in G have distinct degrees (or) no two vertices have same degree.

Defintion 2.7.

Let G = (A, B) denotes a connected IVPNG. Then G is highly irregular interval-valued pythagorean neutrosophic graph (HI-IVPNG) if all vertex is adjacent to the vertices in G have distinct degree.

III. MAIN RESULTS

Theorem 3.1.

Let G be a connected IVNPG and B be a constant function. G is a NEI-IVPNG if and only if G is a NETI-IVPNG.

Proof:
Consider a constant function B. Let B(uv) = ⟨ (M_{B-}(uv),M_{B+}(uv)),(I_{B-}(uv), I_{B+}(uv)),(N_{B-}(uv), N_{B+}(uv)) ⟩ = ⟨ (c_1,c_2),(b_1, b_2),(d_1, d_2) ⟩, ∀ uv ∈ E , where c_1, c_2, b_1,b_2,d_1,d_2 are constants.

Let G be a NEI-IVPNG.
Then, the set of all adjacent edges has distinct degrees.
⇔ d(uv) ≠ d(vw), where E = {uv, vw} are adjacent edges.
⇔ ⟨ d_M(uv), d_I(uv), d_N(uv) ⟩ ≠ ⟨ d_M(vw), d_I(vw), d_N(vw)⟩
⇔ d_M(uv) ≠ d_M(vw) (or) d_I(uv) ≠ d_I(vw) (or) d_N(uv) ≠ d_N(vw) (or) all of them.

Without loss of generality, let us assume that d_M(uv) ≠ d_M(vw)
⇔ (d_{M-}(uv), d_{M+}(uv)) ≠ (d_{M-}(vw), d_{M+}(vw))
⇔ d_{M-}(uv) ≠ d_{M-}(vw) (or) d_{M+}(uv) ≠ d_{M+}(vw) (or) both.
Without loss of generality, let us assume that d_{M-}(uv) ≠ d_{M-}(vw)
⇔ d_{M-}(u) + d_{M-}(v) - 2M_{B-}(uv) ≠ d_{M-}(v) + d_{M-}(w)- 2M_{B-}(vw)
⇔ d_{M-}(u) + d_{M-}(v)- 2c_1 ≠ d_{M-}(v) + d_{M-}(w)- 2c_1
Adding c_1 on both sides,
⇔ d_{M-}(u) + d_{M-}(v)- 2c_1 + c_1 ≠ d_{M-}(v) + d_{M-}(w)- 2c_1 + c_1
⇔ d_{M-}(u) + d_{M-}(v)- c_1 ≠ d_{M-}(v) + d_{M-}(w)- c_1
⇔ d_{M-}(u) + d_{M-}(v)- M_{B-}(uv) ≠ d_{M-}(v) + d_{M-}(w)- M_{B-}(vw)
⇔ td_{M-}(uv) ≠ td_{M-}(vw) ⇔ td_M(uv)⊭ td_M(vw)
⇔ ⟨ td_M(uv), td_I(uv), td_N(uv) ⟩ ≠ ⟨ td_M(vw), td_I(vw), td_N(vw) ⟩
⇔ td(uv) ≠ td(vw), where E= {uv, vw} are adjacent edges.
Then, the set of all adjacent edges has distinct total degrees.

⇔ Hence, G is a NETI-IVPNG.

Remark 3.2.
Let G be a connected IVPNG. If G is both NEI-IVPNG and NETI-IVPNG, then B need not to be a constant function.

Theorem 3.3.
Let G be a connected IVPNG and B is a constant function. If G is a SI-IVPNG, then G is a NEI-IVPNG.

Proof:
Let B(uv) = ⟨ (M_{B-}(uv), M_{B+}(uv)), (I_{B-}(uv),I_{B+}(uv)),(N_{B-}(uv),N_{B+}(uv)) ⟩ = ⟨ (c_1, c_2),(b_1, b_2), (d_1, d_2) ⟩, ∀ uv ∈ E , where c_1, c_2, b_1, b_2, d_1, d_2 are constants. Let uv and vw are any two adjacent edges in G. Assume that G is a SI-IVPNG. Then, all pair of vertices in G having distinct degrees.
⇒ d(u) ≠ d(v) ≠ d(w) ⇒ d(u) ≠ d(w)

Adding d(v) on both sides,
⇒ d(u) + d(v) ≠ d(v) + d(w)
⇒ ⟨ (d_{M-}(u),d_{M+}(u)),(d_{I-}(u),d_{I+}(u)),(d_{N-}(u),d_{N+}(u)) ⟩ + ⟨ (d_{M-}(v),d_{M+}(v)),(d_{I-}(v),d_{I+}(v)), (d_{N-}(v),d_{N+}(v)) ⟩ ≠ ⟨ (d_{M-}(v),d_{M+}(v)),(d_{I-}(v),d_{I+}(v)),(d_{N-}(v),d_{N+}(v)) ⟩ + ⟨ ((d_{M-}(w), (d_{M+}(w)), (d_{I-}(w), d_{I+}(w)),(d_{N-}(w),d_{N+}(w)) ⟩

Adding -2⟨ (c_1, c_2), (b_1, b_2), (d_1, d_2) ⟩ on both sides,
⇒ ⟨ (d_{M-}(u)+d_{M-}(v)-2c_1,d_{M+}(u)+d_{M+}(v)-2c_2),(d_{I-}(u)+ d_{I-}(v)-2b_1,d_{I+}(u)+ d_{I+}(v)- 2b_2),(d_{N-}(u)+ d_{N-}(v)- 2d_1,d_{N+}(u)+ d_{N+}(v)-2d_2) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(w)-2c_1, d_{M+}(v)+ d_{M+}(w)- 2c_2),(d_{I-}(v)+ d_{I-}(w)-2b_1, d_{I+}(v)+ d_{I+}(w)-2b_2),(d_{N-}(v)+ d_{N-}(w)-2d_1, d_{N+}(v)+ d_{N+}(w)- 2d_2) ⟩
⇒ ⟨ (d_{M-}(u)+ d_{M-}(v)-2M_{B-}(uv), d_{M+}(u)+ d_{M+}(v)-2M_{B+}(uv)),(d_{I-}(u)+ d_{I-}(v)-2I_{B-}(uv), d_{I+}(u)+ d_{I+}(v)-2I_{B+}(uv)),(d_{N-}(u)+ d_{N-}(v)-2N_{B-}(uv), d_{N+}(u)+ d_{N+}(v)-2N_{B+}(uv)) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(w)-2M_{B-}(vw), d_{M+}(v)+ d_{M+}(w)-2M_{B+}(vw)), (d_{I-}(v)+ d_{I-}(w)-2I_{B-}(vw), d_{I+}(v)+ d_{I+}(w)-2I_{B+}(vw)), (d_{N-}(v)+ d_{N-}(w)-2N_{B-}(vw), d_{N+}(v)+ d_{N+}(w)-2N_{B+}(vw)) ⟩
⇒⟨(d_{M-}(uv),d_{M+}(uv)),(d_{I-}(uv), d_{I+}(uv)),(d_{N-}(uv), d_{N+}(uv)) ⟩ ≠ ⟨ (d_{M-}(vw),d_{M+}(vw)), (d_{I-}(vw),d_{I+}(vw)),(d_{N-}(vw),d_{N+}(vw)) ⟩
⇒ d(uv) ≠ d(vw)

Hence, the set of all adjacent edges has distinct degree.
This implies that, G is a NEI-IVNG.

Theorem 3.4.
Let G be a connected IVPNG and B is a constant function. If G is a SI-IVPNG, then G is a NETI-IVPNG.

Proof:
Let G be a connected IVPNG and B be a constant function. Suppose that G is SI-IVPNG. By the above theorem, G is a NEI-IVPNG. From theorem 3.1, G is a NETI-IVPNG.

Remark 3.5.
The converse part of the theorem 3.3 and 3.4 need not be true.

Theorem 3.6.
Let G be a connected IVPNG and B denotes a constant function. Then, G is a HI-IVPNG iff G is a NEI-IVPNG.

Proof: Let B(uv) = ⟨ (M_{B-}(uv),M_{B+}(uv)), (I_{B-}(uv), I_{B+}(uv)),(N_{B-}(uv), N_{B+}(uv)) ⟩ = ⟨ (c_1, c_2),(b_1, b_2), (d_1, d_2) ⟩, ∀ uv ∈ E , where c_1, c_2, b_1, b_2, d_1, d_2 are constants.

Consider the adjacent edges uv and vw in G and let G be a HI-IVPNG. Then, by definition 2.7,
⇒ d(u) ≠ d(w)

Adding d(v) on both sides,
⇒ d(u) + d(v) ≠ d(v) + d(w)
⇒ ⟨ (d_{M-}(u),d_{M+}(u)),(d_{I-}(u),d_{I+}(u)),(d_{N-}(u),d_{N+}(u)) ⟩ + ⟨ (d_{M-}(v), d_{M+}(v)),(d_{I-}(v), d_{I+}(v)), (d_{N-}(v), d_{N+}(v)) ⟩ ≠ ⟨ (d_{M-}(v), d_{M+}(v)),(d_{I-}(v), d_{I+}(v)), (d_{N-}(v), d_{N+}(v)) ⟩ + ⟨ (d_{M-}(w), d_{M+} (w)), (d_{I-}(w), d_{I+}(w)),(d_{N-}(w), d_{N+}(w)) ⟩
Adding -2⟨ (c_1, c_2),(b_1, b_2), (d_1, d_2) ⟩ on both sides,
⇒ ⟨ (d_{M-}(u)+ d_{M-}(v)-2c_1, d_{M+}(u)+ d_{M+}(v)-2c_2),(d_{I-}(u)+ d_{I-}(v)-2b_1, d_{I+}(u)+ d_{I+}(v)- 2b_2),(d_{N-}(u)+ d_{N-}(v)-2d_1,d_{N+}(u)+d_{N+}(v)-2d_2) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(w)-2c_1,d_{M+}(v)+d_{M+}(w)- 2c_2),(d_{I-}(v)+ d_{I-}(w)-2b_1, d_{I+}(v)+ d_{I+}(w)-2b_2),(d_{N-}(v)+ d_{N-}(w)-2d_1, d_{N+}(v)+ d_{N+}(w)- 2d_2) ⟩
⇒ ⟨ (d_{M-}(u)+ d_{M-}(v)-2M_{B-}(uv), d_{M+}(u)+ d_{M+}(v)-2M_{B+}(uv)),(d_{I-}(u)+ d_{I-}(v)-2I_{B-}(uv), d_{I+}(u)+ d_{I+}(v)-2I_{B+}(uv)),((u)+ d_{N-}(v)-2N_{B-}(uv), d_{N+}(u)+ d_{N+}(v)-2N_{B+}(uv)) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(w)-2M_{B-}(vw), d_{M+}(v)+ d_{M+}(w)-2M_{B+}(vw)),(d_{I-}(v)+ d_{I-}(w)-2I_{B-}(vw), d_{I+}(v)+ d_{I+}(w)-2I_{B+}(vw)), (d_{N-}(v)+ d_{N-}(w)-2N_{B-}(vw), d_{N+}(v)+ d_{N+}(w)-2N_{B+}(vw)) ⟩
⇒ ⟨ (d_{M-}(uv), d_{M+}(uv)),(d_{I-}(uv), d_{I+}(uv)),(d_{N-}(uv), d_{N+}(uv)) ⟩ ≠ ⟨ (d_{M-}(vw), d_{M+}(vw)),(d_{I-}(vw), d_{I+}(vw)),(d_{N-}(vw), d_{N+}(vw)) ⟩
⇒ d(uv) ≠ d(vw)

Hence, all set of adjacent edges have distinct degree. Therefore, G is a NEI-IVPNG.

Conversely, assume that G is a NEI-IVPNG. All set of adjacent edges in G has distinct degrees. Let v be the vertex which is adjacent to u, w and x. Then we have uv, vw and vx as the adjacent edges in G.
⇒ d(uv) ≠ d(vw) ≠ d(vx)
⇒ ⟨ (d_{M-}(uv), d_{M+}(uv)),(d_{I-}(uv), d_{I+}(uv)),(d_{N-}(uv), d_{N+}(uv)) ⟩ ≠ ⟨ (d_{M-}(vw), d_{M+}(vw)),(d_{I-}(vw), d_{I+}(vw)),(d_{N-}(vw), d_{N+}(vw)) ⟩ ≠ ⟨ (d_{M-}(vx), d_{M+}(vx)),(d_{I-}(vx), d_{I+}(vx)),(d_{N-}(vx), d_{N+}(vx))
⇒ ⟨ (d_{M-}(u)+ d_{M-}(v)-2M_{B-}(uv), d_{M+}(u)+ d_{M+}(v)-2M_{B+}(uv)),(d_{I-}(u)+ d_{I-}(v)-2I_{B-}(uv), d_{I+}(u)+ d_{I+}(v)-2I_{B+}(uv)),(d_{N-}(u)+ d_{N-}(v)-2N_{B-}(uv), d_{N+}(u)+ d_{N+}(v)-2N_{B+}(uv)) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(w)-2M_{B-}(vw), d_{M+}(v)+ d_{M+}(w)-2M_{B+}(vw)),(d_{I-}(v)+ d_{I-}(w)-2I_{B-}(vw), d_{I+}(v)+ d_{I+}(w)-2I_{B+}(vw)), (d_{N-}(v)+ d_{N-}(w)-2N_{B-}(vw), d_{N+}(v)+ d_{N+}(w)-2N_{B+}(vw)) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(x)-2M_{B-}(vx), d_{M+}(v)+ d_{M+}(x)-2M_{B+}(vx)),(d_{I-}(v)+ d_{I-}(x)-2I_{B-}(vx), d_{I+}(v)+ d_{I+}(x)-2I_{B+}(vx)),(d_{N-}(v)+ d_{N-}(x)-2N_{B-}(vx), d_{N+}(v)+ d_{N+}(x)-2N_{B+}(vx)) ⟩

⇒ ⟨ (d_{M-}(u)+ d_{M-}(v)-2c_1, d_{M+}(u)+ d_{M+}(v)-2c_2),(d_{I-}(u)+ d_{I-}(v)-2b_1, d_{I+}(u)+ d_{I+}(v)- 2b_2),(d_{N-}(u)+ d_{N-}(v)-2d_1, d_{N+}(u)+ d_{N+}(v)-2d_2) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(w)-2c_1, d_{M+}(v)+ d_{M+}(w)- 2c_2),(d_{I-}(v)+ d_{I-}(w)-2b_1, d_{I+}(v)+ d_{I+}(w)-2b_2),(d_{N-}(v)+ d_{N-}(w)-2d_1, d_{N+}(v)+ d_{N+}(w)- 2d_2) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(x)-2c_1, d_{M+}(v)+ d_{M+}(x)-2c_2),(d_{I-}(v)+ d_{I-}(x)-2b_1, d_{I+}(v)+ d_{I+}(x)- 2b_2),(d_{N-}(v)+ d_{N-}(x)-2d_1, d_{N+}(v)+ d_{N+}(x)-2d_2) ⟩

Adding 2⟨ (c_1, c_2), (b_1, b_2), (d_1, d_2) ⟩ on both sides,
⇒ ⟨ (d_{M-}(u), d_{M+}(u)),(d_{I-}(u), d_{I+}(u)),(d_{N-}(u), d_{N+}(u)) ⟩ + ⟨ (d_{M-}(v), d_{M+}(v)),(d_{I-}(v), d_{I+}(v)), (d_{N-}(v), d_{N+}(v)) ⟩ ≠ ⟨ (d_{M-}(v), d_{M+}(v)),(d_{I-}(v), d_{I+}(v)),(d_{N-}(v), d_{N+}(v)) ⟩ + ⟨ (d_{M-}(w), d_{M+}(w)), (d_{I-}(w), d_{I+}(w)),(d_{N-}(w), d_{N+}(w)) ⟩ ≠ ⟨ (d_{M-}(v), d_{M+}(v)),(d_{I-}(v), d_{I+}(v)),(d_{N-}(v), d_{N+}(v)) ⟩ + ⟨ (d_{M-}(x), d_{M+}(x)),(d_{I-}(x), d_{I+}(x)),(d_{N-}(x), d_{N+}(x)) ⟩
⇒ d(u) + d(v) ≠ d(v) + d(w) ≠ d(v) + d(x)
⇒ d(u) ≠ d(w) ≠ d(x).
⇒ Every vertex in G is adjacent to the vertices in G, which have distinct degrees.

Hence,G is a HI-IVPNG.

Theorem 3.7.
Let G be a connected IVPNG and B denotes a constant function. Then, G is an HI-IVPNG, iff G is a NETI-IVPNG.

Proof:
Let G be a connected IVPNG. Assume that B is a constant function and G is HI-IVPNG.
⇔ G is a NEI-IVPNG (from above theorem).
⇔ G is a NETI-IVPNG (from Theorem 3.1).

Theorem 3.8.
Let G be a connected IVPNG and B denotes a constant function. If G is a NI-IVPNG, then G is a NEI-IVPNG.

Proof:
Let G be a connected IVPNG and B denotes a constant function. Let B(uv) = ⟨ (M_{B-}(uv), M_{B+}(uv)),(I_{B-}(uv), I_{B+} (uv)),(N_{B-}(uv), N_{B+}(uv)) ⟩ = ⟨ (c_1, c_2), (b_1, b_2), (d_1, d_2) ⟩, ∀ uv ∈ E , where c_1,c_2,b_1,b_2,d_1,d_2 are constants. Consider any two adjacent edges uv and vw in G. Suppose that G is NI-IVPNG.

⇒ The set of all adjacent vertices in G has distinct degree.
⇒ d(u) ≠ d(v) and d(v) ≠ d(w)
⇒ d(u) ≠ d(w)

Adding d(v) on both sides,
⇒ d(u) + d(v) ≠ d(v) + d(w)
⇒ ⟨ (d_{M-}(u), d_{M+}(u)),(d_{I-}(u), d_{I+}(u)),(d_{N-}(u), d_{N+}(u)) ⟩ + ⟨ (d_{M-}(v), d_{M+}(v)),(d_{I-}(v), d_{I+}(v)), (d_{N-}(v), d_{N+}(v)) ⟩ ≠ ⟨ (d_{M-}(v), d_{M+}(v)),(d_{I-}(v), d_{I+}(v)),(d_{N-}(v), d_{N+}(v)) ⟩ + ⟨ (d_{M-}(w), d_{M+}(w)), (d_{I-}(w), d_{I+}(w)),(d_{N-}(w), d_{N+}(w)) ⟩

Subtracting 2⟨ (c_1, c_2), (b_1, b_2), (d_1, d_2) ⟩ on both sides,
⇒ ⟨ d_{M-}(u)+ d_{M-}(v)-2c_1, d_{M+}(u)+ d_{M+}(v)-2c_2),(d_{I-}(u)+ d_{I-}(v)-2b_1, d_{I+}(u)+ d_{I+}(v)- 2b_2),(d_{N-}(u)+ d_{N-}(v)-2d_1, d_{N+}(u)+ d_{N+}(v)-2d_2) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(w)-2c_1, d_{M+}(v)+

d_{M+}(w)- $2c_2$),(d_{I-}(v)+ d_{I-}(w)-2b_1, d_{I+}(v)+ d_{I+}(w)-2b_2),(d_{N-}(v)+ d_{N-}(w)-2d_1, d_{N+}(v)+ d_{N+}(w)- 2d_2)

⇒ ⟨ (d_{M-}(u)+ d_{M-}(v)-2M_{B-}(uv), d_{M+}(u)+ d_{M+}(v)-2M_{B+}(uv)),(d_{I-}(u)+ d_{I-}(v)-2I_{B-}(uv), d_{I+}(u)+ d_{I+}(v)-2I_{B+}(uv)),(d_{N-}(u)+ d_{N-}(v)-2N_{B-}(uv), d_{N+}(u)+ d_{N+}(v)-2N_{B+}(uv)) ⟩ ≠ ⟨ (d_{M-}(v)+ d_{M-}(w)-2M_{B-}(vw), d_{M+}(v)+ d_{M+}(w)-2M_{B+}(vw)),(d_{I-}(v)+ d_{I-}(w)-2I_{B-}(vw), d_{I+}(v)+ d_{I+}(w)-2I_{B+}(vw)), (d_{N-}(v)+ d_{N-}(w)-2N_{B-}(vw), d_{N+}(v)+ d_{N+}(w)-2N_{B+}(vw)) ⟩

⇒ ⟨(d_{M-}(uv),d_{M+}(uv)),(d_{I-}(uv),d_{I+}(uv)),(d_{N-}(uv), d_{N+}(uv)) ⟩ ≠ ⟨ (d_{M-}(vw), d_{M+}(vw)), (d_{I-}(vw), d_{I+}(vw)),(d_{N-}(vw), d_{N+}(vw)) ⟩

⇒ d(uv) ≠ d(vw)

All set of adjacent edges in G has distinct degrees. Therefore, G is a NEI-IVPNG.

Theorem 3.9.
Let G be a connected IVPNG and B is a constant function. If G is a NI-IVPNG, then G is a NETI-IVPNG.

Proof:
Let G be a connected IVPNG. Assume that B is a constant function. Suppose that G is NI-IVPNG.
⇔ G is NEI-IVPNG (from the above theorem).
⇔ G is NETI-IVPNG (from Theorem 3.1).

Remark 3.10.
The converse part of the theorem 3.8 and 3.9 need not be true.

IV. Conclusion

In this article, we explored the neighbourly edge and neighbourly edge totally irregularities on interval valued pythagorean neutrosophic graph and the relation with other irregular property-bearing graphs. Though there are some real-life applications related to IVPNG [7] but neighbourly edge irregularity was not applied till now. In the future, we plan to execute further development with applications in this area.

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