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TRANSMISSION DYNAMICS OF SMOKING- A MATHEMATICAL MODEL

Murugappan Mullai*, G. Madhan Kumar, Grienggrai Rajchakit* and Govindan Vetrivel

Abstract **- Smoking is a disastrous habit that exposes smokers to a greater chance of cardiovascular and blood vessel ailments. Also, it can cause many diseases like cancer, asthma, strokes etc., to smokers. It plays a significant role in one of the economic issues of a country. This article deals with the transmission dynamics of smoking addiction in a country's population. A nonlinear mathematical model is developed here to analyze smoking addiction among the adolescent population. In this model, the four compartments, viz., susceptible male, susceptible female, chain smoker (between male and female) and smokers but not chain smoker (between male and female), are considered to study the transmission dynamics of smoking addiction. The deterministic model for tobacco addiction is formulated to exhibit the smoker's addiction-free equilibrium. Also, the model's reproductive number (R0) is calculated to analyze the spread of the smoking habit. The local stability of these equilibrium points is also analyzed analytically and numerically.**

Keywords— Mathematical Model, Smoking, Equilibrium, Stability.

2010 Mathematics Subject Classification: 34D20, 34D23

I. Introduction

A substance is burned during the smoking process, and the resulting smoke is inhaled, tasted, and absorbed into the bloodstream. Most frequently, the ingredient is dried tobacco leaves packed onto a tiny square of rice paper to form a small, round cylinder known as a "cigarette." Smoking is typically used as a route of administration for recreational drug use because the combustion of dried plant leaves dissipates and disperses the active ingredients into the lungs, where they quickly diffuse into bloodstream circulation and reach human tissue. These compounds are included in an aerosol particle and gas combination, which consists of the pharmacologically active alkaloid nicotine; the vaporization creates heated aerosol and gas, permitting inhalation and deep penetration into the lungs, where the active substances are absorbed into the circulation.

In certain cultures, smoking is used as part of rituals to generate trance-like experiences that participants thought would lead to spiritual enlightenment. Smoking is typically harmful to one's health since inhalation disrupts numerous physiologic systems, such as breathing. Tobacco-related diseases have been demonstrated to kill around half of long-term smokers

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compared to non-smokers' typical mortality rates. From 1990 to 2015, smoking was responsible for nearly five million deaths each year. One of the most common sorts of recreational drug use is smoking. Tobacco smoking is the most common kind, with over one billion individuals worldwide, the vast majority of whom live in developing nations. Cannabis and opium are less prevalent smoking substances. Some of the chemicals are categorized as hard narcotics, such as heroin, although their usage is restricted since they are seldom commercially accessible. Cigarettes are mostly made industrially, although they may also be hand-rolled from loose tobacco and rolling paper. Pipes, cigars, bidis, hookahs, and bongs are among smoking accessories.

Smoking may be traced back to 5000 BCE and has been documented in many different civilizations throughout the globe. In industrialized nations, cigarette smoking is the leading unavoidable risk factor for illness and death. Changes in the prevalence of cigarette smoking in the second half of the twentieth century have lowered current smoking levels to around one-quarter of the adult population, as well as disparities in smoking prevalence and smoking-attributable illnesses between the sexes. Currently, smoking is associated with people of younger age, poorer income, lower educational attainment, and a disadvantaged neighborhood environment. Smokers use cigarettes daily to maintain nicotine levels in the brain, primarily to prevent the unpleasant effects of nicotine withdrawal and adjust their mood. Regular smokers had higher stress and arousal levels than non-smokers and higher impulsivity and neuroticism trait scores.

S. Mishra and M.B. Mishra [6] submitted their review on tobacco and its hazards in the mode of smoking. Sikander et al. [10] framed and discussed a math model regarding the evolution of smoking. R. Charaffedine et al. [2] discussed the gender and educational differences in the association between smoking and health-related issues in Belgium. Olga Perski et al. [7] explained that the motivation to stop smoking could improve the level of non-smokers in the community. In 2019, Ghaus ur Rahman et al. [3] did a mathematical analysis of giving up smoking. I. Todorovic et al. [9] carried out a cross-sectional study with the University of Banja Luka students to check for prevalent cigarette smoking and its factors through statistical sampling. Recently, S.K. Singh et al. [8] did a statistical sample study related to men's tobacco use in India between 2019-2021.

The main objective of this article is to determine a transmission model for smoking addiction through numerical simulation.

II. Methodology

The model is divided into four compartments:

- S_m represents the susceptible male population who may or may not be addicted to the smoking habit.
- S_f denotes the susceptible female population who may or may not be addicted to the smoking habit.
- M represents both male and female population who are smokers but not chain smoker.
- A denotes both male and female population who are chain smokers.

The equations for the model are defined in section III. The parameters of the model are defined in section IV. Numerical simulations obtained through MATLAB software are used to analyze the impact of smoking among different kinds of populations.

III. Model Equations

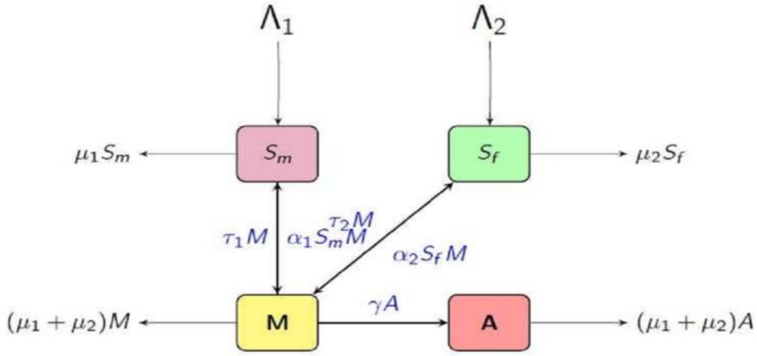
The mathematical model can be framed as follows:

$$\frac{dS_m}{dt} = \Lambda_1 - \alpha_1 S_m M - \mu_1 S_m + \tau_1 M$$
$$\frac{dS_f}{dt} = \Lambda_2 - \alpha_2 S_f M - \mu_2 S_f + \tau_2 M$$
$$\frac{dM}{dt} = \alpha_1 S_m M + \alpha_2 S_f M - \gamma M - (\tau_1 + \tau_2)M - (\mu_1 + \mu_2)M \quad (3.1)$$
$$\frac{dA}{dt} = \gamma M - (\mu_1 + \mu_2)A$$

IV. Description of Parameters

TABLE I. Model Parameters

Parameters	Description
Λ_1	Male recruitment rate
Λ_2	Female recruitment rate
μ_1	Male natural death rate
μ_2	Female natural death rate
α_1 and α_2	Rate of transmission through peer groups of male and female respectively
τ_1 and τ_2	Rate of transmission from M to S_m and S_f
γ	Rate of addicted transmission
$R_0 = \frac{\alpha\Lambda}{\mu(\gamma+\tau+\mu)}$	Basic reproduction number



Λ_1
Λ_2
$\mu_1 S_m$
S_m
S_f
$\mu_2 S_f$
$\tau_1 M$
$\alpha_1 S_m M$
$\tau_2 M$
$\alpha_2 S_f M$
$(\mu_1+\mu_2)M$
M
γA
A
$(\mu_1+\mu_2)A$

V. MALE POPULATION ONLY

Consider $S_f = 0$

$$\frac{dS_m}{dt} = \Lambda_1 - \alpha_1 S_m M - \mu_1 S_m + \tau_1 M$$
$$\frac{dM}{dt} = \alpha_1 S_m M - \gamma M - \tau_1 M - \mu_1 M \quad (5.1)$$
$$\frac{dA}{dt} = \gamma M - \mu_1 A$$

The model has two equilibrium points, namely,

$$I_0 = (S_m^0, M^0, A^0) = (\frac{\Lambda_1}{\mu_1}, 0,0) \quad (5.2)$$

and

$$I_* = (S_m^*, M^*, A^*) \quad (5.3)$$

where,

$$S_m^* = \frac{\gamma+\tau_1+\mu_1}{\alpha_1}$$
$$M^* = \frac{\Lambda_1}{\gamma+\mu_1 - \frac{\mu_1}{\alpha_1}[\frac{\gamma+\tau_1+\mu_1}{\alpha_1(\gamma+\mu_1)}]} \quad (5.4)$$
$$A^* = \frac{\gamma}{\mu_1[\frac{\Lambda_1}{\gamma+\mu_1} - \frac{\mu_1}{\alpha_1}\frac{\gamma+\tau_1+\mu_1}{\alpha_1(\gamma+\mu_1)}]}$$

Theorem 5.1. If R_0 < 1, then the addiction-free equilibrium point exists.

Proof: The evaluated jacobian at addiction-free equilibrium points gives

$$L = \begin{pmatrix} \mu_1 & -\alpha_1\left(\frac{\Lambda_1}{\mu_1}\right) + \tau_1 & 0 \\ 0 & \alpha_1\left(\frac{\Lambda_1}{\mu_1}\right) - \gamma - \tau_1 - \mu_1 & 0 \\ 0 & \gamma & -\mu_1 \end{pmatrix} \quad (5.5)$$

Linearization of the model gives

$$\begin{pmatrix} -\mu_1 - \lambda & -\alpha_1\left(\frac{\Lambda_1}{\mu_1}\right) + \tau_1 & 0 \\ 0 & \alpha_1\left(\frac{\Lambda_1}{\mu_1}\right) - \gamma - \tau_1 - \mu_1 - \lambda & 0 \\ 0 & \gamma & -\mu_1 - \lambda \end{pmatrix} \quad (5.6)$$

The characteristic equation gives

$$(-\mu_1 - \lambda)(\alpha_1\left(\frac{\Lambda_1}{\mu_1}\right) - \gamma - \tau_1 - \mu_1 - \lambda)(-\mu_1 - \lambda) = 0, \quad (5.7)$$

where

$$(-\mu_1 - \lambda) = 0,$$
$$\lambda = -\mu_1$$

$$\alpha_1\left(\frac{\Lambda_1}{\mu_1}\right) - \gamma - \tau_1 - \mu_1 - \lambda = 0$$
$$\lambda = \alpha_1\left(\frac{\Lambda_1}{\mu_1}\right) - \gamma - \tau_1 - \mu_1 \quad (5.8)$$

$$\alpha_1\left(\frac{\Lambda_1}{\mu_1}\right) - \gamma - \tau_1 - \mu_1 < 0$$
$$\alpha_1\left(\frac{\Lambda_1}{\mu_1}\right) < \gamma + \tau_1 + \mu_1$$
$$\frac{\alpha_1\Lambda_1}{\mu_1(\gamma+\tau_1+\mu_1)} < 1$$
$$R_0 < 1.$$

Therefore, the addiction-free equilibrium point exists and R_0< 1.

VI. FEMALE POPULATION ONLY

Consider $S_m = 0$

$$\frac{dS_f}{dt} = \Lambda_2 - \alpha_2 S_f M - \mu_2 S_f + \tau_2 M$$
$$\frac{dM}{dt} = \alpha_2 S_f M - \gamma M - \tau_2 M - \mu_2 M \quad (6.1)$$
$$\frac{dA}{dt} = \gamma M - \mu_2 A$$

The model has two equilibrium points, namely,

$$E_0 = \left(S_f^0, M^0, A^0\right) = (\frac{\Lambda_2}{\mu_2}, 0,0) \quad (6.2)$$

and

$$E_* = (S_f^*, M^*, A^*) \quad (6.3)$$

where,

$$S_f^* = \frac{\gamma+\tau_2+\mu_2}{\alpha_2}$$
$$M^* = \frac{\Lambda_2}{\gamma+\mu_2} - \frac{\mu_2}{\alpha_2}[\frac{\gamma+\tau_2+\mu_2}{\alpha_2(\gamma+\mu_2)}] \quad (6.4)$$
$$A^* = \frac{\gamma}{\mu_2}[\frac{\Lambda_1}{\gamma+\mu_1} - \frac{\mu_1}{\alpha_1}\frac{\gamma+\tau_1+\mu_1}{\alpha_1(\gamma+\mu_1)}]$$

Theorem 6.1. If R_0 < 1, then the addiction-free equilibrium point exists.

Proof: The evaluated jacobian at addiction-free equilibrium points gives

$$L=\begin{pmatrix} \mu_2 & -\alpha_2\left(\frac{\Lambda_2}{\mu_2}\right)+\tau_2 & 0 \\ 0 & \alpha_2\left(\frac{\Lambda_2}{\mu_2}\right)-\gamma-\tau_2-\mu_2 & 0 \\ 0 & \gamma & -\mu_2 \end{pmatrix} \quad (6.5)$$

Linearization of the model gives

$$\begin{pmatrix} -\mu_2-\lambda & -\alpha_2\left(\frac{\Lambda_2}{\mu_2}\right)+\tau_2 & 0 \\ 0 & \alpha_2\left(\frac{\Lambda_2}{\mu_2}\right)-\gamma-\tau_2-\mu_2-\lambda & 0 \\ 0 & \gamma & -\mu_2-\lambda \end{pmatrix} \quad (6.6)$$

The characteristic equation gives

$$(-\mu_2-\lambda)\left(\alpha_2\left(\frac{\Lambda_2}{\mu_2}\right)-\gamma-\tau_2-\mu_2-\lambda\right)(-\mu_2-\lambda)=0, \quad (6.7)$$

where

$$(-\mu_2-\lambda)=0,$$
$$\lambda=-\mu_2$$

$$\alpha_2\left(\frac{\Lambda_2}{\mu_2}\right)-\gamma-\tau_2-\mu_2-\lambda=0$$

$$\lambda=\alpha_2\left(\frac{\Lambda_2}{\mu_2}\right)-\gamma-\tau_2-\mu_2 \quad (6.8)$$

$$\alpha_2\left(\frac{\Lambda_2}{\mu_2}\right)-\gamma-\tau_2-\mu_2<0$$
$$\alpha_2\left(\frac{\Lambda_2}{\mu_2}\right)<\gamma+\tau_2+\mu_2$$

$$\frac{\alpha_2\Lambda_2}{\mu_2(\gamma+\tau_2+\mu_2)}<1$$

$$R_0<1.$$

Therefore, the addiction-free equilibrium point exists and R_0< 1.

VII. Numerical Simulation

The study of the performance of the numerical simulation is carried out to explore the population of males with chain smokers between both males & females and smokers but not chain smokers between both males & females, under addiction-free and endemic equilibrium. Similarly, the calculation can be done for the female population. This study helps us to find the population which is highly and lowly addicted to smoking. For a total of 500 days, the ups and downs of the smoking population can be noted for every interval gap of 50 days. The results show that the male and female population is high when compared to chain smokers and smokers but not chain smokers during addiction-free equilibrium. In the scenario of endemic equilibrium, the chain smoker's population is higher than that of other populations.

The numerical simulation that relates the susceptible male (S_m) with chain smokers (M) and smokers but not chain smokers (A), under addiction-free and endemic equilibrium is shown in Figures 1 and 2.

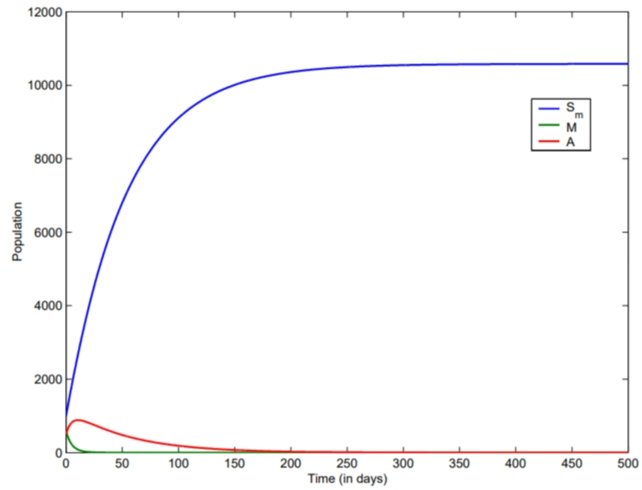


Figure 1. Graph showing the addiction-free equilibrium at $R_0=0.2654<1$

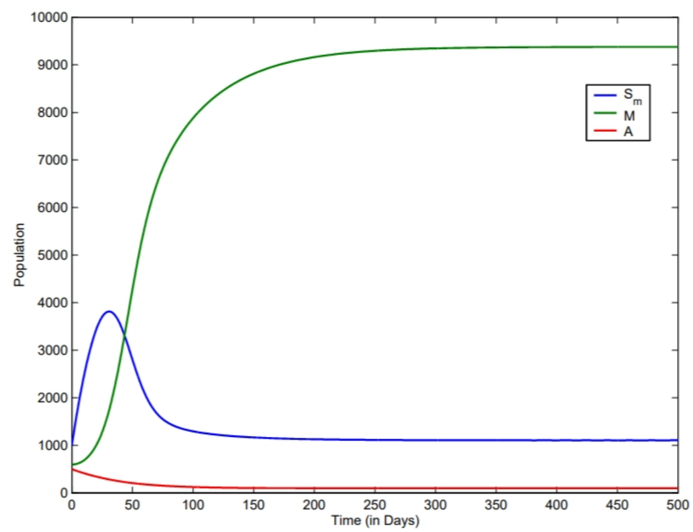


Figure 2. Graph showing the existence of endemic equilibrium at $R_0=1.7234>1$

The numerical simulation that relates the susceptible female (S_f) with chain smokers (M) and smokers but not chain smokers (A), under addiction-free and endemic equilibrium is shown in Figures 3 and 4.

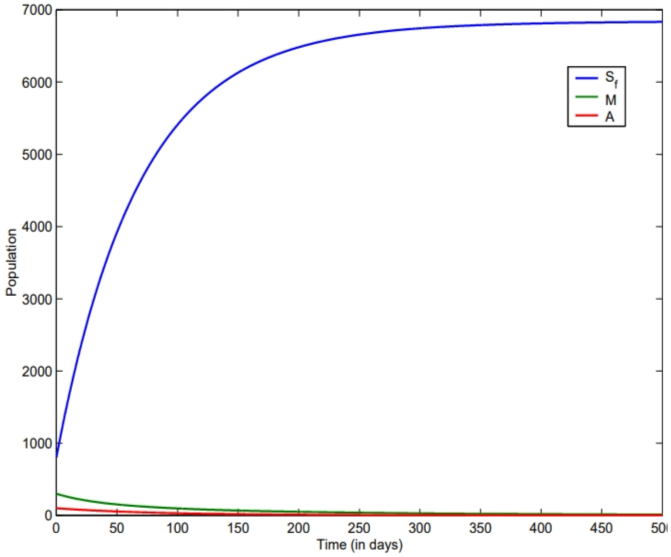


Figure. 3. Graph showing the addiction free equilibrium at $R_0 = 0.6324 < 1$

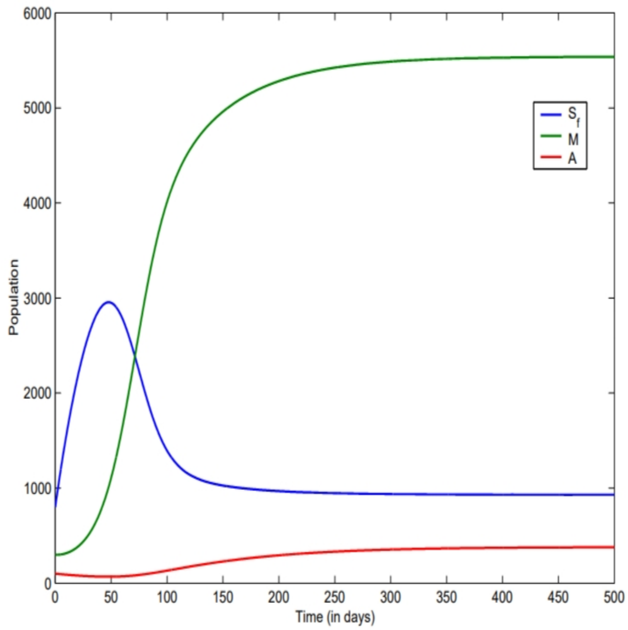


Figure. 4. Graph showing the existence of endemic equilibrium at $R_0 = 2.6344 > 1$

VIII. Conclusion

When R_0 < 1, the smoking population will not exist and for R_0 > 1, the smoking population starts to emerge. Through the numerical simulation, we finalize that the male and female population is much susceptible to smoking addiction during addiction-free equilibrium. Still, in the case of endemic equilibrium, the chain smoker population is high.

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