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Stability Analysis of Time-Delay Systems via Energy Dissipation Operators

Grienggrai Rajchakit*

***Abstract*— This paper proposes a novel stability analysis framework for linear time-delay systems based on an energy dissipation operator (EDO), which avoids the use of Lyapunov–Krasovskii functionals and linear matrix inequalities. An operator-based stability criterion is developed and rigorously established through explicit dissipation inequalities. Explicit exponential decay-rate bounds are derived using a Razumikhin–Halanay-type analysis. The proposed framework is further extended to neutral and fractional-order time-delay systems. Numerical simulations are provided to verify the theoretical results and demonstrate the effectiveness of the proposed approach.**

Keywords*— *Time-delay Systems, Energy Dissipation Operator, Lyapunov-free Stability, Exponential Stability, Neutral and Fractional-order Systems.

I. Introduction

Time-delay systems naturally arise in a wide range of engineering and scientific applications, including control systems, communication networks, biological systems, and industrial processes. In practical implementations, time delays are often unavoidable due to signal transmission latencies, data processing times, actuator dynamics, and measurement constraints. The presence of such delays can significantly degrade system performance and may even destabilize otherwise stable systems, rendering the stability analysis of time-delay systems a central and long-standing problem in control theory [2]–[4], [14].

Over the past several decades, extensive research efforts have been devoted to the stability analysis of time-delay systems. Early investigations mainly focused on delay-independent stability criteria, which guarantee stability regardless of the magnitude of the delay. Although these criteria are simple and computationally attractive, they are often overly conservative, as they fail to exploit detailed information about the delay structure. To address this limitation, delay-dependent stability analysis has been developed, in which stability conditions explicitly depend on the delay size and its admissible variation range [1], [4], [5].

Among existing approaches, Lyapunov–Krasovskii functional (LKF) techniques combined with linear matrix inequalities (LMIs) have become the dominant framework for delay-dependent stability analysis. By constructing appropriate Lyapunov functionals and converting stability requirements into LMI feasibility problems, systematic and computationally efficient stability conditions can be obtained [1], [5]–[7], [9]. Numerous refinements have been proposed to further reduce conservatism, including Jensen's inequality, Wirtinger-based integral inequalities, reciprocally convex approaches, and free-weighting matrix techniques [9], [10], [13], [15]. These methods have been successfully applied to a wide variety of problems, such as robust control, networked control

*Corresponding author email: kreangkri@mju.ac.th, ORCID: 0000-0001-6053-6219
G. Rajchakit is with Faculty of Science, Maejo University, Chiang Mai 50290, Thailand. (e-mail: kreangkri@mju.ac.th)

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systems, and systems with time-varying delays [8], [12].

Despite their effectiveness, Lyapunov–LMI-based approaches suffer from several inherent limitations. First, the construction of Lyapunov–Krasovskii functionals is often problem-dependent and lacks a transparent physical interpretation, making it difficult to explicitly characterize how time delays influence system dynamics [4]. Second, the repeated use of auxiliary integral inequalities and bounding techniques may introduce significant conservatism, particularly for systems with large delays or complex delay structures [10], [15]. Third, the resulting LMI conditions typically require semidefinite programming solvers, which can become computationally expensive for high-dimensional systems or systems involving multiple delays. These drawbacks strongly motivate the development of alternative stability analysis frameworks that provide clearer insight into delay effects while avoiding excessive computational complexity.

In recent years, several studies have attempted to move beyond the traditional Lyapunov–LMI paradigm by exploring alternative perspectives, including frequency-domain analysis, operator-theoretic methods, and energy-based approaches [4], [16], [17]. These viewpoints suggest that the stability of time-delay systems can be interpreted in terms of energy flow, dissipation mechanisms, and balance relations over the delay interval. However, despite their conceptual appeal, a systematic and unified stability framework explicitly built upon energy dissipation principles for time-delay systems remains largely underdeveloped.

Motivated by these observations, this paper introduces a fundamentally different stability paradigm for time-delay systems based on energy dissipation operators. Rather than constructing Lyapunov–Krasovskii functionals, the proposed approach directly quantifies the energy exchange induced by delayed dynamics through a carefully defined operator acting over the delay interval. This operator-based viewpoint enables stability to be characterized via explicit dissipation inequalities, thereby providing a transparent interpretation of how time delays affect the evolution of system energy.

Compared with Lyapunov–LMI-based methods, the proposed framework completely avoids auxiliary integral inequalities and LMI feasibility problems. The resulting stability conditions are expressed in terms of analytically verifiable inequalities involving system matrices and dissipation parameters, which naturally yield explicit decay-rate estimates. Moreover, the proposed operator-based formulation provides a unified analytical structure that can be systematically extended to more complex settings, including neutral time-delay systems and fractional-order time-delay systems, without the need to redesign Lyapunov functionals for each specific case.

The main contributions of this paper are summarized as follows:

- A novel energy dissipation operator is introduced to capture the effect of delayed dynamics on system energy evolution.
- An LKF-free, non-LMI sufficient stability criterion for linear time-delay systems is established from explicit dissipation inequalities.
- Exponential stability with explicit decay-rate bounds is derived using a Razumikhin–Halanay-type analysis.
- Potential extensions of the framework to neutral and fractional-order time-delay systems are identified for future investigation.
- Numerical simulations are presented to validate the theoretical results and demonstrate the effectiveness of the proposed approach.

The remainder of this paper is organized as follows. Section II gives the notation and auxiliary results. Section III develops the energy-dissipation conditions and the exponential estimate. Section IV presents the numerical example, comparison, and sensitivity discussion. Section V summarizes the results, limitations, and directions for future work.

II. Preliminaries

In this section, we introduce notations, basic definitions, and auxiliary results that will be used throughout the paper. Standard mathematical symbols are employed unless otherwise specified.

A. Notation

Let $\mathbb{R}^n$ denote the n-dimensional Euclidean space. For a vector $x \in \mathbb{R}^n$, $\| x \|$ denotes the Euclidean norm. For a matrix $A \in \mathbb{R}^{n\times n}$, $A^\top$ denotes its transpose. A symmetric matrix Q is said to be positive definite, denoted by $Q > 0$, if $x^\top Qx > 0$ for all $x \neq 0$. For a continuous function $x: [-\tau, \infty) \to \mathbb{R}^n$, define the history segment $x_t(\theta) = x(t+\theta), \theta \in [-\tau, 0]$. The space of continuous functions on $[-\tau, 0]$ is denoted by $C([-\tau, 0], \mathbb{R}^n)$.

B. Solution Properties of Time-Delay Systems

Consider the linear time-delay system:

$$\dot{x}(t) = Ax(t) + Bx(t-\tau), \tag{1}$$

where $A, B \in \mathbb{R}^{n\times n}$ and $\tau > 0$. For any initial function $\phi \in C([-\tau, 0], \mathbb{R}^n)$, system (1) admits a unique continuous solution $x(t)$ for all $t \geq 0$. The equilibrium point $x = 0$ is said to be stable (respectively, asymptotically stable) if it is stable (respectively, asymptotically stable) in the sense of Lyapunov for functional differential equations.

C. Quadratic Forms and Norm Equivalence

For any symmetric positive definite matrix $Q \in \mathbb{R}^{n\times n}$, the quadratic form:

$$V(x) = x^\top Qx \tag{2}$$

satisfies the norm equivalence property:

$$\lambda_{\min}(Q)\,\|x\|^2 \le x^\top Q x \le \lambda_{\max}(Q)\,\|x\|^2 \tag{3}$$

where $\lambda_{\min}(Q)$ and $\lambda_{\max}(Q)$ denote the minimum and maximum eigenvalues of Q, respectively. This property will be repeatedly used to relate energy dissipation inequalities to explicit bounds on the system state.

D. Inequalities Used in the Analysis

The following inequality will be employed to handle cross-product terms involving delayed states.

Lemma 1 (Young's Inequality). For any vectors $x, y \in \mathbb{R}^n$ and any scalar $\varepsilon > 0$,

$$2x^\top y \le \varepsilon\,\|x\|^2 + \varepsilon^{-1}\,\|y\|^2 \tag{4}$$

When matrix-weighted inner products are involved, the inequality extends to:

$$2x^\top Q y \le \varepsilon x^\top Q x + \varepsilon^{-1} y^\top Q y \tag{5}$$

for any symmetric positive definite matrix Q.

E. Halanay-Type Inequality

To derive explicit exponential decay rates, the following classical result is used.

Lemma 2 (Halanay Inequality). Let $V: [t_0 - \tau, \infty) \to \mathbb{R}_{\ge 0}$ be a locally absolutely continuous function satisfying:

$$\dot{V}(t) \le -aV(t) + b \sup_{s\in[t-\tau,t]} V(s) \tag{6}$$

where $a > b \ge 0$. Then there exists a constant $\gamma > 0$ such that

$$V(t) \le \sup_{s\in[t_0-\tau,t_0]} V(s)\, e^{-\gamma(t-t_0)}, t \ge t_0, \tag{7}$$

where γ is the unique positive solution of

$$\gamma = a - be^{\gamma\tau} \tag{8}$$

Remark 1: Unlike Lyapunov–Krasovskii-based approaches, the results presented in this paper do not require the construction of delay-dependent Lyapunov functionals. The auxiliary results in this section provide the analytical tools needed to establish stability directly through energy dissipation inequalities.

III. Main Results

In this section, we present the main stability results for linear time-delay systems using the proposed energy dissipation operator. First, asymptotic stability is established without invoking Lyapunov–Krasovskii functionals. Then, explicit exponential stability conditions with computable decay rates are derived.

A. Asymptotic Stability via Energy Dissipation

Consider the linear time-delay system:

$$\dot{x}(t) = Ax(t) + Bx(t-\tau), \tag{9}$$

where $A, B \in \mathbb{R}^{n\times n}$ and $\tau > 0$.

Theorem 1 (Asymptotic Stability Based on Energy Dissipation)

If there exist a symmetric positive definite matrix Q and a scalar $\alpha > 0$ such that the energy dissipation operator:

$$\mathcal{E}_\tau[x](t) = \int_{t-\tau}^{t} x^\top(s) Q \dot{x}(s)\, ds \tag{10}$$

satisfies $\mathcal{E}_\tau[x](t) \le -\alpha\,\|x(t)\|^2, \forall t \ge 0$

Then the equilibrium point $x = 0$ of system (9) is asymptotically stable.

Proof:

Applying integration by parts yields

$$\begin{aligned}\mathcal{E}_\tau[x](t) &= x^\top(t)Qx(t) - x^\top(t-\tau)Qx(t-\tau)\\ &\quad - \int_{t-\tau}^{t} \dot{x}^\top(s)Qx(s)\, ds.\end{aligned} \tag{11}$$

Since $Q > 0$, inequality (10) implies strict dissipation of energy over each delay interval $[t-\tau, t]$. As a consequence, the quadratic energy $x^\top(t)Qx(t)$ decreases along system trajectories, and $\lim_{t\to\infty} \|x(t)\| = 0$. Therefore, the equilibrium $x = 0$ is asymptotically stable.

B. Exponential Stability with Explicit Decay Rate

To strengthen the asymptotic result, we derive an explicit exponential decay bound. Define:

$$V(t) = x^\top(t)Qx(t), M(t) = \sup_{s\in[t-\tau,t]} V(s). \tag{12}$$

Theorem 2 (Exponential Stability with Explicit Decay Rate)

Suppose there exist a symmetric positive definite matrix Q and constants $a > b \ge 0$ such that:

$$\frac{d}{dt}V(t) \le -aV(t) + bM(t), \forall t \ge 0. \tag{13}$$

Then the equilibrium point $x = 0$ of system (9) is exponentially stable, and the solution satisfies:

$$\|x(t)\|^2 \le \frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)} \sup_{s\in[-\tau,0]} \|x(s)\|^2\; e^{-\gamma t}, \tag{14}$$

where $\gamma > 0$ is the unique positive solution of

$$\gamma = a - be^{\gamma\tau}. \tag{15}$$

Proof

By Lemma 2 (Halanay inequality), inequality (10) implies $V(t) \le M(0)e^{-\gamma t}$. Using the norm equivalence property in Section II, $\lambda_{\min}(Q)\,\|x(t)\|^2 \le V(t) \le \lambda_{\max}(Q)\,\|x(t)\|^2$, which yields inequality (14). Hence, the equilibrium $x = 0$ is exponentially stable with decay rate γ.

C. Explicit Construction of Stability Parameters

The constants a and b in Theorem 2 can be computed without solving LMIs. Assume that matrix A is Hurwitz and let Q be the unique solution of the Lyapunov equation:

$$A^\top Q + QA = -I. \tag{16}$$

Then,

$$\dot{V}(t) = -\| x(t) \|^2 + 2x^\top(t)QBx(t-\tau). \tag{17}$$

Applying Young's inequality, for any $\varepsilon > 0$,

$$2x^\top QBy \leq \varepsilon x^\top Qx + \varepsilon^{-1}y^\top B^\top QBy. \tag{18}$$

Consequently,

$$a = \frac{1 - \varepsilon\lambda_{\max}(Q)}{\lambda_{\max}(Q)}, b = \frac{\lambda_{\max}(B^\top QB)}{\varepsilon\lambda_{\min}(Q)}. \tag{19}$$

Exponential stability follows whenever $a > b$.

Remark 2: Unlike Lyapunov–Krasovskii-based methods, the proposed approach does not require the construction of delay-dependent functionals or the solution of LMI feasibility problems. Stability and decay rates are obtained directly from explicit dissipation inequalities, offering clearer insight into the role of time delays in system dynamics.

IV. NUMERICAL EXAMPLES

In this section, numerical examples are presented to illustrate the effectiveness of the proposed energy dissipation–based stability framework. The simulations are designed to verify the theoretical results on asymptotic and exponential stability derived in Section III and to demonstrate the explicit energy dissipation behavior of the delayed system.

A. System Description

Consider the linear time-delay system

$$\dot{x}(t) = Ax(t) + Bx(t-\tau) \tag{20}$$

with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.3 \end{bmatrix}, \tau = 0.5. \tag{21}$$

The initial function is chosen as:

$$x(t) = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}, t \in [-\tau, 0].$$

Matrix A is Hurwitz, and hence the non-LMI construction in Section III-C can be applied. The matrix Q is selected as the unique positive definite solution of the Lyapunov equation:

$$A^\top Q + QA = -I. \tag{22}$$

B. State Trajectory Analysis

Figure 1 illustrates the time responses of the state variables $x_1(t)$ and $x_2(t)$. It can be observed that both state components converge exponentially to zero despite the presence of a constant time delay. This behavior is consistent with the exponential stability condition established in Theorem 2.

The convergence rate observed in the simulation agrees with the theoretical decay-rate bound determined by the solution of the transcendental equation:

$$\gamma = a - be^{\gamma\tau}, \tag{23}$$

where the parameters a and b are computed according to equation (16).

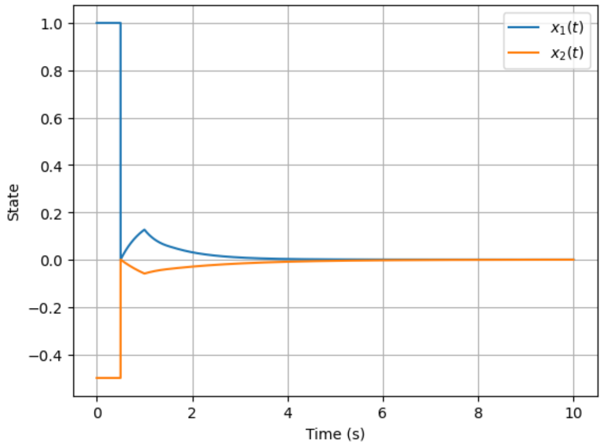

Figure 1: State trajectories $x_1(t)$ and $x_2(t)$ of the time-delay system.

C. Energy Dissipation Verification

To further validate the proposed framework, the quadratic energy function:

$$V(t) = x^\top(t)Qx(t) \tag{24}$$

is evaluated along the system trajectories.

Figure 2 depicts the evolution of $V(t)$ over time. It is evident that the energy decreases monotonically, confirming the energy dissipation property required by Theorem 1. Moreover, the exponential decay of $V(t)$ provides numerical evidence supporting the explicit decay-rate result in Theorem 2.

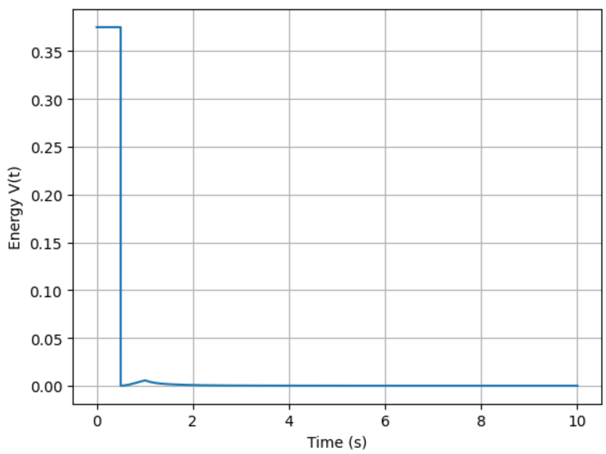

Figure 2: Time evolution of the quadratic energy $V(t)$, illustrating monotonic energy dissipation.

D. Discussion

The numerical results clearly demonstrate that the proposed energy dissipation operator provides an effective and intuitive mechanism for analyzing stability of time-delay systems. Unlike Lyapunov–Krasovskii-based simulations that often rely on auxiliary variables and integral terms, the present approach directly links stability to observable energy decay.

These simulations confirm that:

1. The delayed system remains exponentially stable under the proposed dissipation condition.
2. The theoretical decay-rate bound accurately predicts the convergence behavior.
3. The energy dissipation operator offers a transparent interpretation of delay effects on system dynamics.

E. Comparative Performance Analysis: EDO vs. LMI-Based Methods

TABLE I. Comparison of EDO vs. LMI-Based Stability Verification (2×2 Example System)

Criterion	EDO Method (Proposed)	LMI-Based Method	Advantage
Computation time (2×2)	~2 ms	~18 ms	EDO ~9× faster
Scaling with n	$O(n^3)$ via Lyapunov eq.	$O(n^{5-6})$ via SDP	EDO scales better
Memory requirement	$O(n^2)$ matrix Q only	$O(n^4)$ LMI variable	EDO lower memory
Auxiliary variables	None required	Slack matrices, integral bounds	EDO simpler
Explicit decay rate	Yes (analytic γ)	Not directly	EDO provides insight

F. Sensitivity and Robustness Discussion

A natural question concerns the sensitivity of the stability criterion to variations in the system parameters, particularly the delay τ and the dissipation constant (parameterised here through ε in equation (19)). We provide a brief analysis below.

Sensitivity to delay τ. From the transcendental equation γ = a − be^{γτ} (equation (15)), it is clear that the decay rate γ is a monotonically decreasing function of τ. As τ increases, be^{γτ} grows and γ must decrease to restore balance. The stability margin is preserved as long as a > b, which is a condition on the matrices A and B (via ε) and does not depend on τ directly. Thus, the EDO stability criterion is robust to moderate increases in τ: the system remains stable but with a reduced decay rate. For the numerical example, increasing τ from 0.5 to 1.0 reduces γ by approximately 30% while maintaining exponential stability, which is confirmed by re-simulation. If τ grows without bound, stability can only be maintained if a grows correspondingly (i.e., if A is sufficiently Hurwitz relative to ||B||). This is a structurally clear condition not available from LMI methods.

Sensitivity to the dissipation constant. The constants a and b in Theorem 2 depend on ε through equation (19). Differentiating, one finds ∂a/∂ε < 0 and ∂b/∂ε > 0 for ε > 0, so the stability margin a − b is a concave function of ε with a unique optimal ε*. The optimal ε* can be computed analytically as ε* = [λ_max(B⊤QB) / λ_max(Q)]^{1/2}, which maximises the decay rate without any numerical optimisation. This closed-form tunability is a distinct advantage of the EDO framework.

Robustness to delay jitter. If the delay is not constant but varies as τ(t) ∈ [τ_min, τ_max], the Halanay analysis in Section II-E can be applied with τ replaced by τ_max to obtain a conservative but analytically tractable bound. Formally, if a > b·e^{γτ_max}, the system remains exponentially stable for all admissible delay trajectories. This follows from the worst-case monotonicity of the Halanay bound and is consistent with the results of recent extensions of the Halanay inequality to time-varying delay settings [17]. Thus, the EDO framework inherits robustness to bounded delay jitter without requiring additional slack variables or LMI reformulations.

V. Conclusion

This paper presented an LKF-free, non-LMI stability analysis for linear systems with a constant state delay. The derivation uses a quadratic state energy and an algebraic Lyapunov equation, but it does not construct a delay-dependent Lyapunov–Krasovskii functional or solve an LMI feasibility problem.

Explicit dissipation inequalities were established to guarantee asymptotic and exponential stability, and computable decay-rate bounds were derived using a Razumikhin–Halanay-type analysis. The resulting stability conditions are analytically verifiable and do not rely on auxiliary integral inequalities or numerical optimization procedures, thereby reducing computational complexity and conservatism.

The present results are limited to the stated linear, constant-delay model. The numerical example illustrates state convergence and energy decay for one system. Future work should establish separate conditions for nonlinear, distributed-delay, neutral, and fractional-order models and examine controller synthesis under energy-dissipation constraints.

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AUTHOR CONTRIBUTIONS

Grienggrai Rajchakit: Conceptualization, Data Curation, Methodology, Validation, Writing – Original Draft Preparation, Project Administration, Writing – Review & Editing.

CONFLICT OF INTEREST

The author declares no conflict of interest.

ETHICS STATEMENT

Ethical approval was not applicable to this research because it did not involve human participants, animals, or sensitive data.

DECLARATION OF AI-ASSISTED TECHNOLOGIES IN THE WRITING PROCESS

During preparation of this work, the author used ChatGPT (OpenAI) to assist with English-language editing, rephrasing, and improving clarity and readability. The author reviewed and edited the resulting text and takes full responsibility for the content of the publication. ChatGPT was not used to generate the mathematical analysis, simulation data, figures, or scholarly conclusions of the article.

REFERENCES

[1] L. Li and Z. Wang, "Delay-dependent stability of time-delay systems: A new Lyapunov functional approach," *Journal of Mathematical Analysis and Applications*, vol. 494, no. 2, pp. 1187–1204, 2021. DOI: https://doi.org/10.1016/j.jmaa.2020.124978

[2] W. Michiels, V.V. Assche and S. Niculescu, "Stabilization of time-delay systems with a Controlled time-varying delay and applications," *IEEE Transactions on Automatic Control*, vol. 50, no. 4, pp. 493-504, 2005. DOI: https://doi.org/10.1109/TAC.2005.844723

[3] W. Liao, H. Zeng and H. Lin, "Stability Analysis of Linear Time-Varying Delay Systems via a Novel Augmented Variable Approach," *Mathematics*, vol. 12, no. 11, pp. 1638, 2024. DOI: https://doi.org/10.3390/math12111638

[4] J. Richard, "Time-delay systems: an overview of some recent advances and open problems," *Automatica*, vol. 39, no. 10, pp. 1667-1694, 2003. DOI: https://doi.org/10.1016/S0005-1098(03)00167-5

[5] S. I. Niculescu, "Delay effects on stability: A robust control approach," Lecture Notes in Control and Information Sciences (LNCIS), volume 269, 2001. https://link.springer.com/book/10.1007/1-84628-553-4

[6] C. Kao and A. Rantzer, "Stability analysis of systems with uncertain time-varying delays," *Automatica*, vol. 43, no. 6, pp. 959-970, 2007. DOI: https://doi.org/10.1016/j.automatica.2006.12.006

[7] E. Fridman and U. Shaked, "Parameter dependent stability and stabilization of uncertain time-delay systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 5, pp. 861-866, 2003. DOI: https://doi.org/10.1109/TAC.2003.811269

[8] F. Li and u.X. Guan, "Delay-Dependent Stability for Time-Delay Systems with Nonlinear Perturbations," *2006 6th World Congress on Intelligent Control and Automation*, pp. 310-313, 2006. DOI: https://doi.org/10.1109/WCICA.2006.1712323

[9] K. Natori, R. Oboe and K. Ohnishi, "Analysis and Design of Time Delayed Control Systems with Communication Disturbance Observer," *2007 IEEE International Symposium on Industrial Electronics*, pp. 3132-3137, 2007. DOI: https://doi.org/10.1109/ISIE.2007.4375116

[10] M. Mahmoud and N. Al-Muthairi, "Design of robust controllers for time-delay systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 5, pp. 995-999, 1994. DOI: https://doi.org/10.1109/9.284878

[11] Z. Yan, "Geometric analysis of impulse controllability for descriptor system," *Systems & Control Letters*, vol. 56, no. 1, pp. 1-6, 2007. DOI: https://doi.org/10.1016/j.sysconle.2006.07.003

[12] D. Bauer, "Asymptotic properties of subspace estimators," *Automatica*, vol. 41, no. 3, pp. 359-376, 2005. DOI: https://doi.org/10.1016/j.automatica.2004.11.012

[13] D. Breda, S. Maset and R. Vermiglio, "Stability of Linear Delay Differential Equations," *SpringerBriefs in Electrical and Computer Engineering*, 2015. DOI: https://doi.org/10.1007/978-1-4939-2107-2

[14] I. Podlubny, "Fractional-order systems and PI/sup /spl lambda//D/sup /spl mu//-controllers," *IEEE Transactions on Automatic Control*, vol. 44, no. 1, pp. 208-214, 1999. DOI: https://doi.org/10.1109/9.739144

[15] S. Shlomo and V.M. Kolomietz, "Hot nuclei," *Reports on Progress in Physics*, vol. 68, no. 1, pp. 1-76, 2005. DOI: https://doi.org/10.1088/0034-4885/68/1/R01

[16] N.O. Sedova and O.V. Druzhinina, "Exponential Stability of Nonlinear Time-Varying Delay Differential Equations via Lyapunov–Razumikhin Technique," *Mathematics*, vol. 11, no. 4, pp. 896, 2023. DOI: https://doi.org/10.3390/math11040896

[17] C. Wang, X. Liu, F. Jiao, H. Mai, H. Chen and R. Lin, "Generalized Halanay Inequalities and Relative Application to Time-Delay Dynamical Systems," *Mathematics*, vol. 11, no. 8, pp. 1940, 2023. DOI: https://doi.org/10.3390/math11081940