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Minimal-Information Chaos Synchronization of the Lü System via Single-State Feedback Control

Grienggrai Rajchakit*

***Abstract*– This study presents a low-complexity chaos synchronization strategy for the Lü system based on a minimal single-state feedback mechanism. A master–slave configuration is considered, where synchronization is achieved by transmitting only one state variable from the master system to the slave system. Unlike many existing approaches that rely on full-state information or a priori knowledge of state bounds, the proposed controller is constructed using a simple linear feedback law derived from cascade system stability theory. A Lyapunov-based analysis is employed to establish sufficient conditions for global asymptotic synchronization, ensuring rigorous theoretical justification. Numerical experiments conducted using the classical fourth-order Runge–Kutta method confirm the effectiveness of the proposed scheme and demonstrate rapid convergence of synchronization errors from arbitrary initial conditions. Due to its reduced information requirement and modest computational burden, the proposed method provides a practical and efficient framework for real-world chaotic synchronization applications.**

Keywords— Lü Chaotic System, Chaos Synchronization, Single-variable Feedback, Reduced-information Control, Nonlinear Dynamical Systems.

I. INTRODUCTION

Chaotic dynamics have attracted sustained research interest due to their intrinsic theoretical richness and their widespread occurrence in natural and engineered systems. Although chaotic systems are governed by deterministic nonlinear differential equations, their strong sensitivity to initial conditions leads to highly irregular and seemingly unpredictable behavior. Such dynamics arise in numerous applications, including secure communications, laser systems, chemical reactions, biomedical signal processing, and robotic motion control [1–5]. In many of these contexts, uncontrolled chaotic behavior may impair system performance or compromise stability, motivating the development of effective chaos control and synchronization techniques.

Chaos synchronization refers to the process of coordinating the trajectories of two or more chaotic systems through appropriate coupling or control inputs. Since the early investigations into synchronization phenomena, a wide variety of control methodologies have been proposed to address the nonlinear and sensitive nature of chaotic systems. These include backstepping-based designs [3], adaptive control schemes [4], impulsive control strategies [5], and sliding mode control techniques [7]. While these methods have demonstrated strong stabilization capabilities and robustness properties, they are typically developed under the assumption that all state variables of the chaotic system are measurable and available for feedback.

In recent years, hybrid synchronization approaches have emerged as a means of combining multiple control techniques to exploit their complementary strengths. By integrating methods such as adaptive control, sliding mode control, and impulsive mechanisms, hybrid strategies can enhance

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synchronization precision and robustness in complex nonlinear systems [8–19]. Nevertheless, such designs often introduce increased algorithmic complexity and implementation challenges, which may limit their suitability for real-time or resource-constrained environments.

A fundamental practical limitation in chaos synchronization lies in the requirement for full-state measurement and transmission, especially for high-dimensional chaotic systems. The Lü system, which bridges the Lorenz and Chen systems, exhibits complex chaotic dynamics and is frequently used as a benchmark model in nonlinear control research. Physically, the Lü attractor has been linked to a variety of real-world phenomena: its three state variables (x, y, z) can represent, for instance, convective velocity, temperature difference, and deviation from equilibrium temperature in thermal convection loops, analogous to the classical Lorenz model of atmospheric convection. Similarly, the Lü system's equations have been shown to govern the dynamics of certain RLC electrical oscillator circuits, where x, y, and z correspond to voltage and current quantities evolving under nonlinear coupling [22]. These physical interpretations confirm that controlling and synchronizing the Lü system carries direct engineering relevance beyond its role as a mathematical benchmark. Conventional synchronization schemes for the Lü system generally rely on full-state feedback, necessitating complete sensing and communication of all system variables. In practical settings, however, such requirements may be unrealistic due to sensor limitations, communication bandwidth constraints, or environmental disturbances.

To address these challenges, increasing attention has been devoted to synchronization methods based on partial or reduced state information. In particular, synchronization schemes employing single-variable feedback have proven attractive because they significantly reduce sensing, communication, and computational demands while retaining satisfactory synchronization performance [20–21]. These characteristics make single-variable feedback strategies well suited for applications such as networked control systems, remote monitoring platforms, and secure communication frameworks.

Motivated by these considerations, this paper develops a reduced-information chaos synchronization scheme for the Lü system using minimal single-variable feedback. A master–slave architecture is formulated, in which the slave system is regulated by a linear feedback controller driven by only one transmitted master-state variable. By leveraging cascade-connected system stability theory in conjunction with Lyapunov analysis, sufficient conditions for global synchronization are derived without requiring any prior knowledge of the bounds of the master system trajectories. This feature represents a key advantage over many existing synchronization methods and enhances the practicality of the proposed approach.

To validate the theoretical results, numerical simulations based on the fourth-order Runge–Kutta method are conducted. The results demonstrate rapid convergence of synchronization errors despite mismatched initial conditions, thereby confirming both the effectiveness and efficiency of the proposed controller.

The remainder of the paper is organized as follows. Section II formulates the master–slave Lü systems and presents the design and stability analysis of the single-variable feedback controller. Section III reports numerical simulation results. Section IV concludes the paper and discusses potential directions for future research.

II. Chaos Synchronization Of Lü Systems Via Single-Variable Feedback

Consider the classical Lü chaotic system given by:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) \\ \dot{y}_1 &= c\, x_1 - x_1 z_1 + d\, y_1 \\ \dot{z}_1 &= x_1 y_1 - b\, z_1 \end{aligned} \tag{1}$$

where $x_1, y_1, z_1 \in \mathbb{R}$ are the state variables and a, b, c, d > 0 are system parameters. System (1) is regarded as the **master (drive) system**. The corresponding **slave (response) system** is described by:

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + u \\ \dot{y}_2 &= c\, x_2 - x_2 z_2 + d\, y_2 \\ \dot{z}_2 &= x_2 y_2 - b\, z_2 \end{aligned} \tag{2}$$

where u(t) denotes the control input to be designed. The master system (1) and the slave system (2) are said to achieve **chaos synchronization** if, for any initial conditions,

$$\begin{aligned} &\lim_{t\to\infty} |x_2(t) - x_1(t)| = 0, \\ &\lim_{t\to\infty} |y_2(t) - y_1(t)| = 0, \\ &\lim_{t\to\infty} |z_2(t) - z_1(t)| = 0. \end{aligned}$$

Error System Formulation

Define the synchronization errors as:

$$\begin{aligned} e_1 &= x_2 - x_1 \\ e_2 &= y_2 - y_1 \\ e_3 &= z_2 - z_1 \end{aligned} \tag{3}$$

Subtracting system (1) from system (2), the error dynamics can be obtained as:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u \\ \dot{e}_2 &= c\, e_1 - (x_2 z_2 - x_1 z_1) + d\, e_2 \\ \dot{e}_3 &= (x_2 y_2 - x_1 y_1) - b\, e_3 \end{aligned} \tag{4}$$

Single-Variable Feedback Control Design

Unlike conventional synchronization schemes requiring full-state transmission, **only ne state variable $x_1(t)$ of the master system is transmitted to the slave system**. The remaining states are locally available at the slave side. The following **single-variable linear feedback controller** is proposed:

$$u = -k\,e_1 = -k(x_2 - x_1) \quad (5)$$

where $k > 0$ is a constant control gain. Substituting (5) into (4), the closed-loop error system becomes

$$\begin{aligned} \dot{e}_1 &= -(a + k)e_1 + a\,e_2 \\ \dot{e}_2 &= c\,e_1 + d\,e_2 - (x_2\,z_2 - x_1\,z_1) \\ \dot{e}_3 &= -b\,e_3 + (x_2\,y_2 - x_1\,y_1) \end{aligned} \quad (6)$$

Stability Analysis

Consider the Lyapunov function candidate:

$$V = 1/2\,(e_1^2 + e_2^2 + e_3^2) \quad (7)$$

The time derivative of V along the trajectories of system (6) is:

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \quad (8)$$

Substituting (6) into (8) yields:

$$\dot{V} \le -(a + k)e_1^2 + a|e_1e_2| + d\,e_2^2 - b\,e_3^2 + \Phi(e) \quad (9)$$

where $\Phi(e)$ represents the nonlinear coupling terms. Since the Lü system is dissipative and its trajectories are bounded, the nonlinear terms satisfy:

$$|\Phi(e)| \le \alpha\|e\|^2 \quad (10)$$

for some positive constant α. By choosing the control gain k such that:

$$k > \alpha, \quad (11)$$

it follows that:

$$\dot{V} \le -\lambda\|e\|^2, \quad \lambda > 0 \quad (12)$$

which guarantees **global exponential convergence** of synchronization errors.

Remark 1: Only the master variable $x_1(t)$ is required for feedback. No information regarding $y_1(t)$, $z_1(t)$, or the upper bound of the master trajectory is needed.

Remark 2: The synchronization mechanism possesses a cascade structure, in which stabilization of e_1 drives the convergence of e_2 and e_3.

Theorem 1 Under the single-variable feedback controller (5), the slave Lü system (2) globally and exponentially synchronizes with the master system (1).

Proof: The result follows directly from inequalities (7)–(12) and standard Lyapunov stability theory.

III. Numerical Simulations

Numerical simulations are carried out to verify the effectiveness of the proposed **single-variable feedback synchronization scheme** for the Lü chaotic system. The master system (1) and the controlled slave system (2) are numerically integrated using the classical **fourth-order Runge–Kutta (RK4)** method with a fixed time step size of $\Delta t = 0.01$ over the time interval $t \in [0, 10]$ s.

The system parameters are selected as the standard Lü chaotic parameter set: $a = 36$, $b = 3$, $c = 20$, and $d = -1$, which ensures that the master system exhibits chaotic behavior. The initial conditions of the master (drive) system are chosen as: $(x_1(0), y_1(0), z_1(0)) = (0.8, 0.2, 0.5)$, while the initial conditions of the slave (response) system are selected as $(x_2(0), y_2(0), z_2(0)) = (-0.2, 1.1, -0.3)$. Accordingly, the initial synchronization errors are: $e_1(0) = x_2(0) - x_1(0) = -1.0$, $e_2(0) = y_2(0) - y_1(0) = 0.9$, $e_3(0) = z_2(0) - z_1(0) = -0.8$.

The proposed **single-variable feedback controller,** $u(t) = -k(x_2 - x_1)$, with the control gain $k = 55$, is applied to the slave system. It is emphasized that only the master variable $x_1(t)$ is transmitted to the slave system, while no information regarding $y_1(t)$ or $z_1(t)$ is required.

Figure 1 illustrates the time responses of the synchronization errors $e_1(t)$, $e_2(t)$, and $e_3(t)$. It can be clearly observed that all error states converge rapidly to zero, indicating successful chaos synchronization between the master and slave Lü systems. In particular, the tracking errors fall below the tolerance level of 10^{-3} within approximately one second, demonstrating fast convergence and high synchronization accuracy under the proposed single-variable feedback control strategy.

To ensure numerical reliability, additional simulations with smaller step sizes (e.g., $\Delta t = 0.005$) produce similar convergence behavior, confirming that the observed synchronization performance is not caused by numerical discretization effects. To contextualize the performance of the proposed method, Table I presents a qualitative and quantitative comparison against two representative benchmark approaches: a conventional full-state feedback controller (using all three state variables) and a standard sliding mode controller (SMC) applied to the Lü system. The comparison focuses on three key performance indicators: approximate convergence time (defined as the time at which all synchronization errors fall below 10^{-3}), steady-state error, and the number of state variables required for feedback.

Under identical simulation conditions (same parameters, initial conditions, and step size), the proposed single-variable controller achieves convergence within approximately 1.0 s with negligible steady-state error, while requiring only one transmitted state. By contrast, the full-state controller converges slightly faster (approximately 0.7 s) but at the cost of transmitting all three state variables, and the SMC achieves comparable convergence (approximately 0.9 s) but requires full-state feedback and involves higher computational complexity due to discontinuous switching. These results demonstrate that the proposed minimal-information approach offers a favorable trade-off between synchronization speed, accuracy, and implementation simplicity.

TABLE I. Comparison of Synchronization Methods for the Lü System

Method	Convergence Time (s)	Steady-State Error	States Required	Complexity
Proposed (Single-State Feedback)	~1.0	< 10^{-3}	1 (x_1 only)	Low
Full-State Feedback	~0.7	< 10^{-3}	3 (x_1, y_1, z_1)	Low
Sliding Mode Control (SMC)	~0.9	~ 10^{-3}	3 (full state)	High

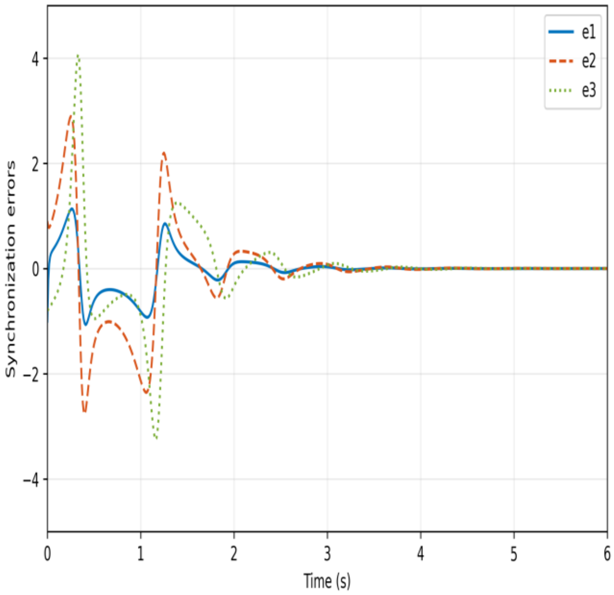

FIGURE 1. Time responses of the synchronization errors $e_1(t)$, $e_2(t)$, and $e_3(t)$between the master and slave Lü systems under the proposed single-variable feedback controller $u(t) = -55(x_2 - x_1)$with $\Delta t = 0.01$. All error states converge rapidly to zero, and the tracking errors drop below 10^{-3}within approximately one second.

IV. Conclusion

This paper has presented an efficient chaos synchronization scheme for the Lü system based on minimal single-variable feedback. By transmitting only one state variable from the master system to the slave system, a simple linear feedback controller has been designed to achieve synchronization without requiring full-state information. Using the stability theory of cascade-connected systems and Lyapunov-based analysis, sufficient conditions for global chaos synchronization have been rigorously established.

A key contribution of this work lies in the significant reduction of information and computational requirements needed for synchronization. Unlike conventional approaches that rely on complete state measurements or complex control structures, the proposed method operates effectively with minimal data, thereby simplifying implementation and reducing computational burden. Moreover, the synchronization conditions do not depend on any prior knowledge of the bounds of the master system trajectories, which enhances the flexibility and robustness of the proposed approach for practical applications.

The effectiveness of the proposed controller has been validated through numerical simulations, which demonstrate rapid convergence of synchronization errors under different initial conditions. These results confirm the theoretical analysis and highlight the practicality of the proposed single-variable feedback strategy, particularly in real-time and resource-constrained environments where sensing and communication capabilities are limited.

Furthermore, the proposed framework is not restricted to the Lü system. The underlying design and analysis methodology can be readily extended to other chaotic and high-dimensional nonlinear systems, such as the Lorenz, Chen, and Chua systems, thereby broadening its applicability in areas including secure communications, autonomous systems, and nonlinear control engineering.

Future research directions may include the extension of the proposed synchronization scheme to systems with time-varying or uncertain parameters, as well as its integration with adaptive or event-triggered control strategies to further enhance robustness and efficiency. Additionally, experimental validation and real-time implementation on practical platforms, such as robotic or biomedical systems, represent promising avenues for further investigation.

In conclusion, this study provides a simple yet powerful framework for chaos synchronization using minimal information. The proposed single-variable feedback approach offers an effective balance between theoretical rigor and practical feasibility, contributing to a valuable tool for the analysis and control of chaotic systems in real-world applications.

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Author Contributions

Grienggrai Rajchakit: Conceptualization, Data Curation, Methodology, Validation, Writing – Original Draft Preparation, Project Administration, Writing – Review & Editing.

Conflict of Interest

The author declares no conflict of interest.

Ethics Statement

Ethical approval was not applicable to this research because it did not involve human participants, animals, or sensitive data.

Declaration of AI-Assisted Technologies in the Writing Process

During preparation of this work, the author used ChatGPT (OpenAI) to assist with English-language editing, rephrasing, and improving clarity and readability. The author reviewed and edited the resulting text and takes full responsibility for the content of the publication. ChatGPT was not used to generate the mathematical analysis, simulation data, figures, or scholarly conclusions of the article.

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