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# First Zagreb Index and its Characteristics on Neutrosophic Graph

Govindan Vetrivel, Murugappan Mullai*, Grienggrai Rajchakit*, Meyyappan Sangavi and R. Surya

***Abstract* – Topological indices mark an irreplaceable place for applications in crisp and fuzzy graphs. These indices are extended to the neutrosophic graphical idea to rectify the imprecise values or information acquired before, since the uncertain cases are organized and allocated as a separate membership called "indeterminacy". We apply and explore the First zagreb index and its properties on the neutrosophic graphical system in the line of Wiener and Forgotten indices. This fills the gap between fuzzy and its graphical extensions on indices discussion, thereby extends the applicable areas. Also, an improvised and unique application is portrayed to observe the importance of First zagreb index in the neutrosophic theme of graphs. This contributes to the real life in a greater way than the fuzzy idea.**

*Keywords—Topological Index, Zagreb Index of First Type, Neutrosophic Set, Neutrosophic Graph, Neutrosophic Zagreb Index of First Type, Neutrosophic Entire Zagreb Index of First Type.*

**TABLE 1. Abbreviations Used.**

| Description | Abbreviation |
|---|---|
| Topological Indices | ToIn |
| Fuzzy Graphs | FuGr |
| Intuitionistic Fuzzy Graph | InFuGr |
| Neutrosophic Set | NeuS |
| Neutrosophic Graph | NeuGr |
| First Zagreb Index | Fi-ZaI |
| Neutrosophic Subgraph | NeuSG |
| Partial Neutrosophic Subgraph | PaNeuSG |
| First Entire Zagreb Index | FIENZag |
| Membership Assignment Value | MAV |
| Minimal Spanning Tree | MiST |

## I. INTRODUCTION

Graph theory is a significant area for dealing with world scenarios and applications. Major solutions for problems are attained using the structural and characteristic properties of graph theory and its variants. More work is being done on the labeling, domination, coloring, etc., in the background of crisp & fuzzy graph theory. The crisp set concept refinement by introducing fuzzy-based set theory [1] has occurred to improve the accuracy. This new set theory recreated graphs with fuzziness [2]. Later, the fuzzy-based graphical approach [3] is explored to attain basic and essential results. A new idea of implementing the ToIn in the fuzzy graphical setting [4] is then carried out. The accomplishment of ToIn in the bipolar perspective of

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FuGr [5] is processed. Also, the Wiener absolute and Randic indexes in bipolar FuGr [6-7] are rendered. The process of executing the F-index [8-9] and edge F-index [10] in the FuGr background was then put forth. In addition, the treatment of the FuGr with other indices like first & second Zagreb index [11-12], Hyper Wiener index [13], Hyper-Zagreb index [14], etc., continued to be explored. A clear discussion about the randic and harmonic index [15] of fuzzy type with brief results on bounds and based product operations has been done. This randic index is used further to determine a criminal with a more serious crime record. The First entire zagreb index in various FuGr [16] is learnt, and their isomorphic relations are investigated. This idea is used to enhance the effectiveness of internet systems. Also, the researchers recently introduced the Gutman index to the FuGr setting [17], where the relationship of this index with other FuGr types is analyzed, and a human trafficking application is provided. Further, a profound investigation of some properties of the F-index in bipolar FuGr [18] and their relationship with other indices is conducted. An application to find a compatible life partner and gene elements is produced with clarity. Then, a comprehensive analysis with indices of crisp & fuzzy type using the formulae [19] is generated. The examination of various ToIn in fuzzy random graphs and the relation between the union and intersection operations of ToIn of fuzzy random graphs [20] is conducted.

Due to incomplete result value, there is a need for an improved fuzzy set theory that has been introduced with the non-membership function. Also, it is named as "Intuitionistic Fuzzy Set" [21]. Later, the construction of the InFuGr [22], which segregates the importance of false events, was established. Some irreplaceable works exemplify the results and applications on InFuGr [23], which was worked on as a team, constituting a significant understanding and involvement of other researchers to participate in the fuzzy-based research. In terms of indices, the Wiener index on InFuGr [24] is demonstrated, and the same is true of the connectivity index. An application to ensure a good water supply through the pipeline is portrayed. The second Zagreb and somber index of the third & fourth versions in InFuGr structure [25] generate some essential outcomes. This study helps them to formulate an Internet routing application that increases the internet speed. The versions of somber indices in InFuGr [26] are derived with a framed formula. It is used to test the performance level of vaccination centers during the pandemic. An application on transport network flow by implying the Wiener index in InFuGr of rough type [27] is pointed out and explained.

An observation is made on the uncertain conditions in the previous studies, and it helped to declare a new set theory concept called NeuS [28-29], that explicitly organizes the uncertain events from the application point of view. A graphical representation using this neutrosophic set is figured out, and known as a NeuGr [30]. This set-based theory postulation was refined and inaugurated in the new name: single-valued neutrosophic graphs [31]. A glimpse of ToIn types is elaborately listed, and the Wiener index in NeuGr [32] is established in detail. The presentation based on the degree & total degree indices in several NeuGr [33] is displayed. The forgotten index in the NeuGr environment [34] has flourished and is being applied to view among the households that efficiently use electricity. The connectivity indices idea is exposed to the NeuGr [35] background that supports applying it in computer and transport network systems.

This manuscript encloses the investigation on Fi-ZaI and Fi-ZaI of the entire type newly in the NeuGr setting. Some crucial findings related to subgraph and isomorphic properties are also analyzed with an appropriate application.

The section I provides a brief introduction on graph theory and their extension to FuGrs. This section I also records the fuzzy and intuitionistic fuzzy based works on ToIn and our observation of other index types in NeuGr. Section II is given to list the definitions of NeuS and NeuGr, which is quite useful to apply ToIn on neutrosophic background. The foundation for Fi-ZaI on NeuGr is laid and some theoretical properties are discussed in the Section III. Section IV shows the algorithm to be followed to find the Fi-ZaI. A glance on FIENZag on NeuGr is illustrated in the Section V with the definition and theorems. Section VI elaborates the applicability and reliability of Fi-ZaI in the neutrosophic graphical setting.

## II. PRELIMINARIES

This section encloses the basic terminologies and definitions to picturize our proposed model.

**Definition 2.1.** [28]

Consider the universe Z. A NeuS $\overline{\mathrm{N}}$ framed on Z is called $\overline{\mathrm{R}} = \{(\mathrm{t}, \mathrm{T}_{\overline{N}}(\mathrm{t}), \mathrm{I}_{\overline{\mathrm{N}}}(\mathrm{t}), \mathrm{F}_{\overline{\mathrm{N}}}(\mathrm{t})) : \mathrm{t} \in \mathrm{X}\}$, where $\mathrm{T}_{\overline{\mathrm{N}}}(\mathrm{t}) : \mathrm{Z} \to [0,1]$, $\mathrm{I}_{\overline{\mathrm{N}}}(\mathrm{t}) : \mathrm{Z} \to [0,1]$, $\mathrm{F}_{\overline{\mathrm{N}}}(\mathrm{t}) : \mathrm{Z} \to [0,1]$ are said to be functions for truth(available), indeterminacy(unsure) and false(unavailable) membership of t on $\overline{\mathrm{N}}$ respectively and it satisfies the condition $0 \leq \mathrm{T}_{\overline{\mathrm{N}}} + \mathrm{I}_{\overline{\mathrm{N}}} + \mathrm{F}_{\overline{\mathrm{N}}} \leq 3, \forall\, \mathrm{t} \in \mathrm{Z}$.

**Definition 2.2.** [31]

A NeuGr is mentioned as $\mathrm{G} = (\ddot{\mathrm{Y}}, \alpha, \beta)$, where $\alpha = (\mathrm{T}_{\ddot{a}}, \mathrm{I}_{\ddot{a}}, \mathrm{F}_{\ddot{a}})$ and $\beta = (\mathrm{T}_{\mathrm{b}}, \mathrm{I}_{\mathrm{b}}, \mathrm{F}_{\mathrm{b}})$ and holds the following conditions,

(i) Let $\mathrm{T}_{\ddot{a}} : \ddot{\mathrm{Y}} \to [0,1]$, $\mathrm{I}_{\ddot{a}} : \ddot{\mathrm{Y}} \to [0,1]$ & $\mathrm{F}_{\ddot{a}} : \ddot{\mathrm{Y}} \to [0,1]$ denote the available, unsure, & unavailable memberships of the element $\mathrm{a_i} \in \ddot{\mathrm{Y}}$, respectively and $0 \leq \mathrm{T}_{\ddot{a}}(\mathrm{a_i}) + \mathrm{I}_{\ddot{a}}(\mathrm{a_i}) + \mathrm{F}_{\ddot{a}}(\mathrm{a_i}) \leq 3$, for all $\mathrm{a_i} \in \ddot{\mathrm{Y}}$.

(ii) Let $\mathrm{T}_{\mathrm{b}} : \varepsilon \subseteq \ddot{\mathrm{Y}} \times \ddot{\mathrm{Y}} \to [0,1]$, $\mathrm{I}_{\mathrm{b}} : \varepsilon \subseteq \ddot{\mathrm{Y}} \times \ddot{\mathrm{Y}} \to [0,1]$ and $\mathrm{F}_{\mathrm{b}} : \varepsilon \subseteq \ddot{\mathrm{Y}} \times \ddot{\mathrm{Y}} \to [0,1]$ denote the available (1), unsure (2), and unavailable (3) memberships of $(\ddot{\mathrm{u}}_\mathrm{i}, \ddot{\mathrm{u}}_\mathrm{j})$ edge respectively, such that

$$\mathrm{T}_{\mathrm{b}}(\mathrm{a_i}, \mathrm{a_j}) \leq \mathrm{low}[\mathrm{T}_{\ddot{a}}(\mathrm{a_i}), \mathrm{T}_{\ddot{a}}(\mathrm{a_j})] \quad (1)$$

$$\mathrm{I}_{\mathrm{b}}(\mathrm{a_i}, \mathrm{a_j}) \leq \mathrm{low}[\mathrm{I}_{\ddot{a}}(\mathrm{a_i}), \mathrm{I}_{\ddot{a}}(\mathrm{a_j})] \quad (2)$$

$$\mathrm{F}_{\mathrm{b}}(\mathrm{a_i}, \mathrm{a_j}) \leq \mathrm{high}[\mathrm{F}_{\ddot{a}}(\mathrm{a_i}), \mathrm{F}_{\ddot{a}}(\mathrm{a_j})] \quad (3)$$

and $0 \leq \mathrm{T}_{\mathrm{b}}(\mathrm{a_i}, \mathrm{a_j}) + \mathrm{I}_{\mathrm{b}}(\mathrm{a_i}, \mathrm{a_j}) + \mathrm{F}_{\mathrm{b}}(\mathrm{a_i}, \mathrm{a_j}) \leq 3$, for every edge $(\mathrm{a_i}, \mathrm{a_j})$.

## III. FIRST ZAGREB INDEX AND ITS PROPERTIES ON NEUTROSOPHIC GRAPHS

**Definition 3.1**.

Consider $\text{Ǥ} = (\ddot{\Upsilon}, \alpha, \beta)$ as a NeuGr. A Fi-ZaI of the NeuGr Ǥ is represented by $\text{ZINGr}_1(\text{Ǥ})$ (4) and is defined as,

$\text{ZINGr}_1(\text{Ǥ}) = \sum_{a\in\ddot{\Upsilon}} \left(Ʈ_{ä}(a), Ɨ_{ä}(a), Ƒ_{ä}(a)\right)(d^2(a))$
$= \sum_{a\in\ddot{\Upsilon}} (Ʈ_{ä}(a), Ɨ_{ä}(a), Ƒ_{ä}(a))(d^2_{Ʈ_{ä}}(a), d^2_{Ɨ_{ä}}(a), d^2_{Ƒ_{ä}}(a))$
$= \sum_{a\in\ddot{\Upsilon}} (Ʈ_{ä}(a)d^2_{Ʈ_{ä}}(a) + Ɨ_{ä}(a)d^2_{Ɨ_{ä}}(a) + Ƒ_{ä}(a)d^2_{Ƒ_{ä}}(a))$
$=\sum_{a\in\ddot{\Upsilon}} Ʈ_{ä}(a)d^2_{Ʈ_{ä}}(a) + \sum_{v\in\Upsilon} Ɨ_{ä}(a)d^2_{Ɨ_{ä}}(a) + \sum_{a\in\ddot{\Upsilon}} Ƒ_{ä}(a)d^2_{Ƒ_{ä}}(a)$ (4)

(i.e.), $\text{ZINGr}_1(\text{Ǥ}) = Ʈ\text{ZINeuGr}_1(\text{Ǥ}) + Ɨ\text{ZINGr}_1(\text{Ǥ}) + Ƒ\text{ZINGr}_1(\text{Ǥ})$

**Definition 3.2**.

Consider $\text{Ǥ} = (\ddot{\Upsilon}, \alpha, \beta)$ as a NeuGr. The modified First zagreb index of the NeuGr Ǥ is denoted by $\text{ZagNeuGr}_1(\text{Ǥ})$ (5) and is defined by

$\text{ZagNeuGr}_1(\text{Ǥ}) = \sum_{a\in\ddot{\Upsilon}} (Ʈ_{ä}(a), Ɨ_{ä}(a), Ƒ_{ä}(a))\ d(a))^2$
$=\sum_{\ddot{u}\in\ddot{\Upsilon}} [Ʈ_{ä}(a)(d_{Ʈ_{ä}}(a))]^2 + \sum_{\ddot{u}\in\ddot{\Upsilon}} [Ɨ_{ä}(a)(d_{Ɨ_{ä}}(a))]^2 + \sum_{\ddot{u}\in\ddot{\Upsilon}} [Ƒ_{ä}(a)(d_{Ƒ_{ä}}(a))]^2$ (5)

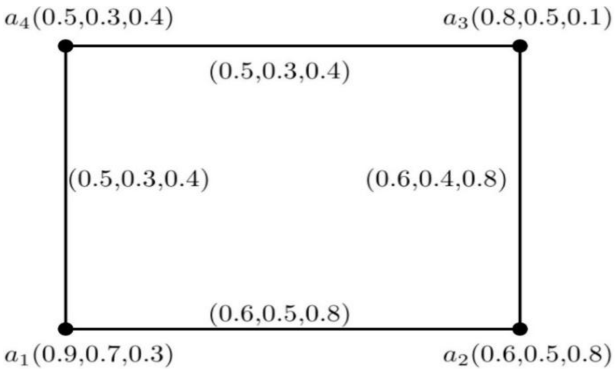


**FIGURE 1. Neutrosophic graph with $\text{ZagNeuGr}_1(\text{Ǥ})$ =7.507**

**Theorem 3.3.**

Consider Ǥ be a NeuGr with h vertices & g edges. Then

(i) $\text{ZagNeuGr}_1(\text{Ǥ}) \le h^2(Ł^2 + ḭ^2 + f^2)$
(ii) $\text{ZagNeuGr}_1(\text{Ǥ}) \le 12h^2g^2$

**Proof:**

(i) By the conditions $Ʈ_{ä}(a) \le 1$, $Ɨ_{ä}(a) \le 1$, $Ƒ_{ä}(a) \le 1$, the following outcome is received:

$$\text{ZagNeuGr}_1(\text{Ǥ}) = [\sum_{a\in\Upsilon} [Ʈ_{ä}(a)(d_{T_1}(a))]^2 + \sum_{v\in\Upsilon} [Ɨ_{ä}(a)(d_{Ɨ_{ä}}(a))]^2 + \sum_{a\in\Upsilon} [Ƒ_{ä}(a)(d_{Ƒ_{ä}}(a))]^2]$$
$$\le [[\sum_{a\in\Upsilon} Ʈ_{ä}(a)]^2[\sum_{a\in\Upsilon} (d_{Ʈ_{ä}}(a))]^2 + [\sum_{a\in\Upsilon} Ɨ_{ä}(a)]^2[\sum_{a\in\Upsilon} (d_{Ɨ_{ä}}(a))]^2 + [\sum_{a\in\Upsilon} Ƒ_{ä}(a)]^2[\sum_{a\in\Upsilon} (d_{Ƒ_{ä}}(a))]^2]$$
$$\le h^2(Ł^2 + ḭ^2 + f^2)$$

(ii) By subdivision (i) and the result $(Ł, ḭ, f)(\text{Ǥ}) = (2\sum Ʈ_2, 2\sum Ɨ_2, 2\sum Ƒ_2) \le (2g, 2g, 2g)$. The required inequality is as follows,

$$\text{ZagNeuGr}_1(\text{Ǥ}) \le h^2(Ł^2 + ḭ^2 + f^2) \le h^2[(2g)^2 + (2g)^2 + (2g)^2] \le 12h^2g^2.$$

**Definition 3.4.**

Consider $\text{Ǥ} = (\ddot{\Upsilon}, \alpha, \beta)$ be a NeuGr and $a \in \ddot{\Upsilon}$. Then the $\text{ZagNeuGr}_1(\text{Ǥ})$ at the vertex a of the NeuGr Ǥ is denoted by $\text{ZagNeuGr}_1(a)$ (6):

$\text{ZagNeuGr}_1(a) = [\text{ZagNeuGr}_1(\text{Ǥ}) - \text{ZagNeuGr}_1(\text{Ǥ}_a)]$ (6)

**Proposition 3.5.**

Consider $Ⱨ=(\ddot{\Upsilon}', \alpha', \beta')$ be a PaNeuSG of a NeuGr $\text{Ǥ} = (\ddot{\Upsilon}, \alpha, \beta)$. Then, $\text{ZagNeuGr}_1(Ⱨ) \le \text{ZagNeuGr}_1(\text{Ǥ})$

**Proof:**

As Ⱨ is a PaNeuSG of Ǥ, we say $\alpha'(a) \le \alpha(a)$ and $\beta'(ab) \le \beta(ab)$ for any a,b $\in \ddot{\Upsilon}'$. This implies, $d_{Ⱨ}(a) = \sum_{b\in\ddot{\Upsilon}'} \beta'(ab) \le \sum_{b\in\ddot{\Upsilon}} \beta(ab) = d_{\text{Ǥ}}(a)$.

Therefore,

$$\text{ZagNeuGr}_1(Ⱨ) = \sum_{a\in\ddot{\Upsilon}'} (Ʈ'_{ä}(a), Ɨ'_{ä}(a), Ƒ'_{ä}(a))\ d(a))^2 \le \sum_{a\in\ddot{\Upsilon}} (Ʈ_{ä}(a), Ɨ_{ä}(a), Ƒ_{ä}(a))\ d(a))^2 = \text{ZagNeuGr}_1(\text{Ǥ})$$

It shows that, $\text{ZagNeuGr}_1(Ⱨ) \le \text{ZagNeuGr}_1(\text{Ǥ})$

**Corollary 3.6.**

Consider $Ⱨ=(\ddot{\Upsilon}', \alpha', \beta')$ as a NeuSG of a NeuGr $\text{Ǥ} = (\ddot{\Upsilon}, \alpha, \beta)$. Then, $\text{ZagNeuGr}_1(Ⱨ) \le \text{ZagNeuGr}_1(\text{Ǥ})$

**Theorem 3.7.**

Consider a NeuGr Ǥ and take $0 \le s_1 \le s_2 \le 1$. Then,
$\text{ZagNeuGr}_1(\text{Ǥ}^{s_2}) \le \text{ZagNeuGr}_1(\text{Ǥ}^{s_1})$.

**Proof:**

Since $\text{Ǥ}^{s_2}$ is a PaNeuSG of $\text{Ǥ}^{s_1}$, then by proposition 3.6, the result $\text{ZagNeuGr}_1(\text{Ǥ}^{s_2}) \le \text{ZagNeuGr}_1(\text{Ǥ}^{s_1})$ is true.

**Corollary 3.8.**

Consider a NeuGr $G$ and take $0 \leq s_1 \leq s_2 \ldots \leq s_n \leq 1$. This implies that, $\mathrm{ZagNeuGr}_1(G^{s_n}) \leq \mathrm{ZagNeuGr}_1(G^{s_{n-1}}) \leq \cdots \leq \mathrm{ZagNeuGr}_1(G^{s_2}) \leq \mathrm{ZagNeuGr}_1(G^{s_1})$.

**Theorem 3.9.**

Consider $G = (\ddot{Y}, \alpha, \beta)$ be a NeuGr. Then $\mathrm{ZagNeuGr}_1(G) \leq \mathrm{ZagNeuGr}_1(C[G])$, where $C[G]$ denotes the completion NeuGr.

**Proof:**
Since $C[G]$ is a completion NeuGr of $G$, $\beta(ab) \leq \beta^C(ab)$, for any ab belongs to the edge set. This implies, $G$ is a PaNeuSG of $C[G]$. Therefore, the result follows by the proposition 3.6.

**Corollory 3.10.**

$\mathrm{ZagNeuGr}_1(G) = d(d-1)^2$, for any vertex d of NeuGr $G$.

**Theorem 3.11.**

Consider the NeuGrs $G_1$ and $G_2$ to be isomorphic. Therefore, $\mathrm{ZagNeuGr}_1(G_1) = \mathrm{ZagNeuGr}_1(G_2)$

**Proof:**
Since the graphs $G_1$ & $G_2$ are isomorphic NeuGrs, there exist a bijection $\theta$ from $\ddot{Y}_1$ to $\ddot{Y}_2$ and $\alpha_1(a) = \alpha_2(\theta(a))$ & $\beta_1(ab) = \beta_2(\theta(a), \theta(b))$, for all a, b $\in \ddot{Y}_1$.

Therefore,
$d_{G_1}(b) = \sum_{a \in \ddot{Y}_1} \beta_2(\theta(a), \theta(b)) = \sum_{\theta(a) \in \ddot{Y}_2} \beta_2(\theta(a), \theta(b)) = d_{G_2}(\theta(b))$

Thus,

$\mathrm{ZagNeuGr}_1(G_1) = \sum_{b \in \ddot{Y}_1} [\alpha_2(\theta(b) d_{G_2}(\theta(b))]^2 = \sum_{\theta(b) \in \ddot{Y}_2} [\alpha_2(\theta(b) d_{G_2}(\theta(b))]^2 = \mathrm{ZagNeuGr}_1(G_2)$

## IV. Algorithm

An algorithm is given in this section to understand the steps to calculate the first zagreb index of NeuGr.

**Step 1:** Find the degree of all vertices in $G$ using d(a)= $\sum_{b \in \ddot{Y}} \beta(a, b)$

**Step 2:** Build the NeuSG $G_b$ for each b$\in\ddot{Y}$.

**Step 3:** Calculate the $\mathrm{ZagNeuGr}_1$ in a neutrosophic graph by $\mathrm{ZagNeuGr}_1 = \sum_{\ddot{u} \in \ddot{Y}} (\mathrm{T}_{\ddot{a}}(a), \mathrm{I}_{\ddot{a}}(a), \mathrm{F}_{\ddot{a}}(a))\, d(a))^2$

**Step 4:** The $\mathrm{ZagNeuGr}_1$ of a vertex a of the NeuGr is obtained by using,
$\mathrm{ZagNeuGr}_1(a) = [\mathrm{ZagNeuGr}_1(G) - \mathrm{ZagNeuGr}_1(G_a)]$

## V. First Entire Zagreb Index on Neutrosophic Graphs

**Definition 5.1.**

Let $G = (\ddot{Y}, \alpha, \beta)$ notifies a NeuGr. Then the FIENZag of NeuGr $G$ is represented by $\mathrm{EZagNeuGr}_1(G)$ (7) and is given by

$$\mathrm{EZagNeuGr}_1(G) = \sum_{a \in \ddot{Y}} (\mathrm{T}_{\ddot{a}}(a), \mathrm{I}_{\ddot{a}}(a), \mathrm{F}_{\ddot{a}}(a))\, d(a))^2 + \sum_{e \in \ddot{Y} x \ddot{Y}} (\mathrm{T}_{b}(e), \mathrm{I}_{b}(e), \mathrm{F}_{b}(e))\, d(e))^2 \quad (7)$$

where the vertex is represented by 'a' and 'b' denotes the edge.

**Theorem 5.2.**

Consider a NeuGr $G$ with h vertices and g edges. Then $\mathrm{EZagNeuGr}_1 \leq h^2 Tot_a^2(G) + g^2 Tot_e^2(G)$, where $Tot_a^2(G)$ and $Tot_e^2(G)$ denotes the total degree of vertices and total degree of edges respectively.

**Proof:**
The FIENZag of $G$ is defined as, $\mathrm{EZagNeuGr}_1 = \sum_{a \in \ddot{Y}} (\mathrm{T}_{\ddot{a}}(a), \mathrm{I}_{\ddot{a}}(a), \mathrm{F}_{\ddot{a}}(a))\, d(a))^2 + \sum_{e \in \ddot{Y} x \ddot{Y}} (\mathrm{T}_{b}(e), \mathrm{I}_{b}(e), \mathrm{F}_{b}(a))\, d(e))^2$

This implies that,
$\leq \sum_{a \in \ddot{Y}} (\mathrm{T}_{\ddot{a}}(a), \mathrm{I}_{\ddot{a}}(a), \mathrm{F}_{\ddot{a}}(a))^2 \sum_{a \in \ddot{Y}} (d(a))^2 + \sum_{e \in \ddot{Y} x \ddot{Y}} (\mathrm{T}_{b}(e), \mathrm{I}_{b}(e), \mathrm{F}_{b}(e))^2 \sum_{e \in \ddot{Y} x \ddot{Y}} (d(e))^2$

$\leq (\sum_{a \in \ddot{Y}} (\mathrm{T}_{\ddot{a}}(a), \mathrm{I}_{\ddot{a}}(a), \mathrm{F}_{\ddot{a}}(a)))^2 (\sum_{a \in \ddot{Y}} d(a))^2 + (\sum_{e \in \ddot{Y} x \ddot{Y}} (\mathrm{T}_{b}(e), \mathrm{I}_{b}(e), \mathrm{F}_{b}(e)))^2 (\sum_{e \in \ddot{Y} x \ddot{Y}} d(e))^2$

$\leq (\sum_{a \in \ddot{Y}} (\mathrm{T}_{\ddot{a}}(a), \mathrm{I}_{\ddot{a}}(a), \mathrm{F}_{\ddot{a}}(a)))^2\, Tot_a^2(G) + (\sum_{e \in \ddot{Y} x \ddot{Y}} (\mathrm{T}_{b}(e), \mathrm{I}_{b}(e), \mathrm{F}_{b}(e)))^2\, Tot_e^2(G)$

Since each membership function of a vertex and edge of a NeuGr lies between 0 and 1, we have

$\mathrm{EZagNeuGr}_1 \leq h^2 Tot_a^2(G) + g^2 Tot_e^2(G)$

**Definition 5.3.**
Consider $G = (\ddot{Y}, \alpha, \beta)$ as a NeuGr and $a \in \ddot{Y}$. Then the $\mathrm{EZagNeuGr}_1(G)$ (8) at the vertex a of the NeuGr $G$ is expressed as:

$\mathrm{EZagNeuGr_1(a)} = [\mathrm{EZagNeuGr_1(G)} - \mathrm{EZagNeuGr_1(G_a)}]$ (8)

**Theorem 5.4.**

Consider a NeuGr H by deleting an edge from G. Then $\mathrm{EZagNeuGr_1(H)} \leq \mathrm{EZagNeuGr_1(G)}$

**Proof:**

Since H=$(\ddot{Y}', \alpha', \beta')$ is a graph obtained by the removal of an edge from the NeuGr G = $(\ddot{Y}, \alpha, \beta)$, the vertex MAV is equal for both the graphs and also it is equal for edges if it bears both E and E'.

Therefore, the relationship between the MAVs of graphs G & H is $\alpha(a) \geq \alpha'(a)\ \forall\, a$ and $\beta(e) \geq \beta'(e)\ \forall e$.

Thus, the degree d(a) ≥ d'(a) $\forall\, a$ and d(e) ≥ d'(e) $\forall$e, where d & d' denotes the degree of G & H respectively.

We know that,

$\mathrm{EZagNeuGr_1(G)} = \sum_{a\in \ddot{Y}(G)} (T_{\ddot{a}}(a), I_{\ddot{a}}(a), F_{\ddot{a}}(a))\, d(a))^2 + \sum_{e\in \ddot{Y}x\ddot{Y}(G)} (T_b(e), I_b(e), F_b(e))\, d(e))^2$

$\geq \sum_{a\in \ddot{Y}(G)} \left(T'_{\ddot{a}}(a), I'_{\ddot{a}}(a), F'_{\ddot{a}}(a))\, d'(a)\right)^2 + \sum_{e\in \ddot{Y}x\ddot{Y}(G)} (T'_b(e), I'_b(e), F'_b(e))\, d'(e))^2$

$= \sum_{a\in \ddot{Y}(H)} \left(T'_{\ddot{a}}(a), I'_{\ddot{a}}(a), F'_{\ddot{a}}(a))\, d'(a)\right)^2 + \sum_{e\in \ddot{Y}x\ddot{Y}(H)} (T'_b(e), I'_b(e), F'_b(e))\, d'(e))^2$

$= \mathrm{EZagNeuGr_1(H)}$

This shows that, $\mathrm{EZagNeuGr_1(H)} \leq \mathrm{EZagNeuGr_1(G)}$

**Theorem 5.5.**

Consider a NeuGr H by removing a vertex from G. Therefore, $\mathrm{EZagNeuGr_1(H)} \leq \mathrm{EZagNeuGr_1(G)}$

**Proof:**

Since H=$(\ddot{Y}', \alpha', \beta')$ is a graph obtained by the removal of a vertex from the NeuGr G = $(\ddot{Y}, \alpha, \beta)$, $\alpha(a) = \alpha'(a)$ if a $\in \ddot{Y} \cap \ddot{Y}'$. If not, then $\alpha(a) > \alpha'(a)$. Similarly for edges, $\beta(e) = \beta'(e)$ if $e \in (\ddot{Y} \times \ddot{Y}) \cap (\ddot{Y}' \times \ddot{Y}')$. If not, then $\beta(a) > \beta'(a)$.

Therefore, the relationship between the MAVs of graphs G and H is $\alpha(a) \geq \alpha'(a)\ \forall\, a$ and $\beta(e) \geq \beta'(e)\ \forall e$.

Thus, the degree d(a) ≥ d'(a) $\forall\, a$ and d(e) ≥ d'(e) $\forall$e, where d & d' denotes the degree of G & H respectively.

We know that,

$\mathrm{EZagNeuGr_1(G)} = \sum_{a\in \ddot{Y}(G)} (T_{\ddot{a}}(a), I_{\ddot{a}}(a), F_{\ddot{a}}(a))\, d(a))^2 + \sum_{e\in \ddot{Y}x\ddot{Y}(G)} (T_b(e), I_b(e), F_b(e))\, d(e))^2$

$\geq \sum_{a\in \ddot{Y}(G)} \left(T'_{\ddot{a}}(a), I'_{\ddot{a}}(a), F'_{\ddot{a}}(a))\, d'(a)\right)^2 + \sum_{e\in \ddot{Y}x\ddot{Y}(G)} (T'_b(e), I'_b(e), F'_b(e))\, d'(e))^2$

$= \sum_{a\in \ddot{Y}(H)} \left(T'_{\ddot{a}}(a), I'_{\ddot{a}}(a), F'_{\ddot{a}}(a))\, d'(a)\right)^2 + \sum_{e\in \ddot{Y}x\ddot{Y}(H)} (T'_b(e), I'_b(e), F'_b(e))\, d'(e))^2$

$= \mathrm{EZagNeuGr_1(H)}$

This shows that, $\mathrm{EZagNeuGr_1(H)} \leq \mathrm{EZagNeuGr_1(G)}$

**Theorem 5.6.**

Consider a NeuSG H of the NeuGr G. Then $\mathrm{EZagNeuGr_1(H)} \leq \mathrm{EZagNeuGr_1(G)}$

**Proof:**

As H= $(\ddot{Y}', \alpha', \beta')$ represents a subgraph obtained from the NeuGr G = $(\ddot{Y}, \alpha, \beta)$, therefore $\alpha(a) \geq \alpha'(a)\ \forall\, a$ and $\beta(e) \geq \beta'(e)\ \forall e$.

Thus, the degree d(a) ≥ d'(a) $\forall\, a$ and d(e) ≥ d'(e) $\forall$e, where d & d' denotes the degree of G and H respectively.

$\mathrm{EZagNeuGr_1(G)} = \sum_{a\in \ddot{Y}(G)} (T_{\ddot{a}}(a), I_{\ddot{a}}(a), F_{\ddot{a}}(a))\, d(a))^2 + \sum_{e\in \ddot{Y}x\ddot{Y}(G)} (T_b(e), I_b(e), F_b(e))\, d(e))^2$

$\geq \sum_{a\in \ddot{Y}(G)} \left(T'_{\ddot{a}}(a), I'_{\ddot{a}}(a), F'_{\ddot{a}}(a))\, d'(a)\right)^2 + \sum_{e\in \ddot{Y}x\ddot{Y}(G)} (T'_b(e), I'_b(e), F'_b(e))\, d'(e))^2$

$= \sum_{a\in \ddot{Y}(H)} \left(T'_{\ddot{a}}(a), I'_{\ddot{a}}(a), F'_{\ddot{a}}(a))\, d'(a)\right)^2 + \sum_{e\in \ddot{Y}x\ddot{Y}(H)} (T'_b(e), I'_b(e), F'_b(e))\, d'(e))^2$

$= \mathrm{EZagNeuGr_1(H)}$

This shows that, $\mathrm{EZagNeuGr_1(H)} \leq \mathrm{EZagNeuGr_1(G)}$

**Theorem 5.7.**

Consider a MiST H of the NeuGr G. Then $\mathrm{EZagNeuGr_1(H)} \leq \mathrm{EZagNeuGr_1(G)}$

**Proof:**

Given: H is a MiST of NeuGr G. Then, H is a NeuSG of G. Therefore, from theorem 5.6, we say $\mathrm{EZagNeuGr_1(H)} \leq \mathrm{EZagNeuGr_1(G)}$.

**Theorem 5.8.**

Consider the NeuGrs $Ǥ_1$ & $Ǥ_2$, that are isomorphic to one another. Then, $\text{EZagNeuGr}_1(Ǥ_1) = \text{EZagNeuGr}_1(Ǥ_2)$

**Proof:**

The bijective map $\theta$ from $Ÿ_1$ to $Ÿ_2$ exist here, since NeuGrs $Ǥ_1$ and $Ǥ_2$ are isomorphic to each other. Then, $\forall a, b \in Ÿ_1$, we have $\alpha_1(b) = \alpha_2(\theta(b))$ and $\beta_1(ab) = \beta_2(\theta(a)\theta(b))$.

Thus,

$d_{Ǥ_1}(b) = \sum_{a \in Ÿ_1} \beta_1(ab) = \sum_{\theta(a) \in Ÿ_2} \beta_2(\theta(a)\theta(b)) = d_{Ǥ_2}(\theta(b))$

This implies that,

$$\text{EZagNeuGr}_1(Ǥ) = \sum_{b \in Ÿ(Ǥ_1)} \left(Ʈ_{ä}(b), Ɨ_{ä}(b), Ƒ_{ä}(b)\right) d_{Ǥ_1}(b)\Big)^2 + \sum_{ab \in Ÿ x Ÿ(Ǥ_1)} \left(Ʈ_{ƀ}(ab), Ɨ_{ƀ}(ab), Ƒ_{ƀ}(ab)\right) d_{Ǥ_1}(ab)\Big)^2$$

$$= \sum_{b \in Ÿ(Ǥ_2)} \left(Ʈ_{ä}(\theta(b)), Ɨ_{ä}(\theta(b)), Ƒ_{ä}(\theta(b))\right) d_{Ǥ_2}(\theta(b))\Big)^2 + \sum_{\theta(a)\theta(b) \in Ÿ x Ÿ(Ǥ_2)} \begin{pmatrix} Ʈ_{ƀ}(\theta(a)\theta(b)), Ɨ_{ƀ}(\theta(a)\theta(b)), \\ Ƒ_{ƀ}(\theta(a)\theta(b))) \\ d_{Ǥ_2}(ab) \end{pmatrix}^2$$

$= \text{EZagNeuGr}_2(Ǥ)$

Therefore,

$\text{EZagNeuGr}_1(Ǥ) = \text{EZagNeuGr}_2(Ǥ)$.

## VI. Application

The calculation is found to solve the decision-making problems. With the alternatives and attributes, the best alternative is chosen by constructing the NeuGr with the set of vertices as the alternative set and the set of edges as the relationship between them. The parameter selection can be made with a clear representation in NeuGr compared to fuzzy graph theory, since the discussion is carried out in 3-logic-based neutrosophic theory. Through the application presented in [11], the demonstration for in NeuGr is equipped by choosing the parameters with neutrosophic memberships. The following case study based on Multicriteria Decision-Making shows the applicability of our proposed model. Suppose a teacher wants to select the class leader from the set of students (alternatives) $S_1$, $S_2$, $S_3$ and $S_4$ based on the parameters (attributes) $P_1$, $P_2$, $P_3$, $P_4$, $P_5$ and $P_6$. The parameters are considered as $P_1$ = Marks, $P_2$= Hardwork, $P_3$ = Extra Curricular Activity, $P_4$ = Discipline, $P_5$ = Obedient, and $P_6$ = Helping mind. The above said parameter values will not be the same for all students, but a good leader can be selected under these circumstances. A development of NeuGrs by assigning each membership of vertices and edges is initially important. Corresponding to each parameter, the construction of NeuGrs will be carried out, where the vertex set components denote the students of the class and the edge between vertices represent the relation of influence. The score value of each student and the score value of influence between the students must be calculated first. Based on the score values and the weight of parameters, the NeuGrs should be constructed to find the Fi-ZaI of each student and their normalized score. Finally, the alternative's order is arranged to observe the best student for leadership.

## VII. Conclusion

In this article, the zagreb index of first type and the entire zagreb index of first type are introduced in the NeuGr structure, and some comparable indices properties of MiST & isomorphism with normal subgraph are highlighted with some theorems in the neutrosophic setting. Our current work is used to establish the Fi-ZaI in the neutrosophic setting, thereby the results and application are achieved to compare the reliability with the past fuzzy works. Our future work is to explore the other ToIn in the neutrosophic environment and to illustrate their features and applications.


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## Author contributions

Govindan Vetrivel: Conceptualization, Data Curation, Methodology, Validation, Writing – Original Draft Preparation;

Murugappan Mullai: Project Administration, Writing – Review & Editing;

Grienggrai Rajchakit: Project Administration, Supervision, Writing – Review & Editing;

Meyyappan Sangavi: Project Administration, Supervision, Writing – Review & Editing;

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No conflict of interests was disclosed.

## Ethics Statements

This research did not involve human participants, animal subjects, or sensitive personal data, and therefore did not require ethical approval.

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