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# Convertible Bond Pricing in Chinese Transportation Industry : A Comparison Methods Between Binomial Tree model and Black-Scholes Model

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**Abstract**

One bond type that can lower financing costs for the issuer is a convertible bond. Additionally, its characteristic with option value gives investors access to high-yielding, low-drawdown, and superior investment instruments. Exactly, convertible bonds have a strong market appeal to investors. In recent years, the issuance scale of convertible bonds has continued to expand, and its share in the bond market has gradually increased. Fair pricing is essential to maintaining the convertible bond market's smooth operation. In light of this, the convertible bonds in the transportation sector listed on the Shanghai Stock Exchange are chosen for this article. Following the acquisition of the fundamental data pertaining to convertible bonds, the bonds are fitted into the bond list using the Black-Scholes and Binomial Tree models. The theoretical value is then priced empirically after other pertinent factors have been duly taken into account. Comparing the estimation with their actual values to obtain the efficiency results, which indicates that Black-Scholes model yields a more accurate estimation than any Binomial Tree model with preset step sizes. The holistic undervaluation means the favorable sentiments of investors towards it. In summary, the contribution of pricing projects to the operation of underlying industries and the economy boost inspired.

**Keywords**: Convertible Bonds Pricing, Binomial Tree Model, Black-Scholes Model, Transportation Industry

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## 1.0 Introduction

Convertible bonds are special financial products between ordinary corporate bonds and stock financing tools. Their uniqueness is reflected in the embedded call option in the product design, which is a particular corporate bond that can be converted into common stock under certain conditions after a period of issuance. Therefore, this product has the triple attributes of debt, equity and options and is sought after by issuers and investors.

Numerous convertible bond pricing techniques have emerged since convertible bonds were first introduced to the US market in 1843 (Jiang, Zhao, & Yang, 2012). China issued its first convertible bond in 1992, but it did not start researching convertible bond pricing until 1998 (Han et al., 2012). Just 82 convertible bonds total of 274.2 billion yuan, were issued on the Chinese stock market between 2006 and 2016. In comparison to the mainstream financing channel bond market, the convertible bond market is small and growing slowly, with no obvious financing advantages. After the implementation of new regulations on refinancing and the reduction of holdings in the Chinese market in 2017, convertible bonds gradually gained popularity and became a favorite in the securities issuance market (Liu, 2018). For one thing, the China Securities Regulatory Commission started to tighten funding due to recent changes in domestic regulatory rules. Convertible bond financing, for another, is a significant substitute product for private placement and is not restricted by financing frequency. A unique possibility for development has been brought about by convertible bonds, the market heat has continued to soar. China's annual convertible bond market has tremendously exploded throughout the five years from 2017 to 2021, being a new channel for direct financing of listed enterprises.

In spite of the expansion of convertible bonds, China's convertible bond market continues to face serious obstacles, including listed companies' unjustifiable financing practices, the imbalanced distribution of equity and bond financing, and their limited utilization of cutting-edge financial derivatives for financing (Wu, 2011). These financing structure issues of corporations can be partially resolved by expanding the convertible bond market, enhancing the convertible bond system, and resolving the practical application challenges of convertible bonds. At the same time, the lower financing costs of issuers can also give investors the opportunity to make profits from exercising conversion rights so that both the industrial and financial economy can develop more vigorously and sustainably.

Influenced by the early enlightenment and the advanced research of related disciplines, the research on convertible bond pricing abroad is relatively mature. Nowadays, the methods with high international recognition are based on the B-S model of random process and differential equation analysis, the binomial or ternary tree model based on the discrete state tree of stock prices and interest rates to simulate the convertible bond pricing process, or the Monte Carlo simulation method based on the optimal stopping theory to consider the pricing of convertible bonds with complex option structures like Longstaff-Schwartz model. However, the development of convertible bonds in China is out of stage with theoretical research, and the types of convertible bonds offered are not rich enough. Domestic scholars' research on market pricing stops at policy impact, issuer credit rating, weak efficient market and investor sentiment factors. It rarely involves the fluctuation of the underlying stock price, interest rate fluctuation, dividend payment, redemption and resale. Consequently, a pricing model that reflects the conditions of the Chinese domestic market can be created by taking lessons from the pricing theories and practices integrating with the real circumstances.

According to data released by the Ministry of Transport (2020), as of the end of 2019, China's railway, highway, shipping and port business scale ranked first in the world. Chungeng Wu, the director of the Policy Research Office and spokesperson of the Ministry of Transport of China, stated that China has become a strong transportation country, and transportation has entered a new era of high-quality development (China Economic Summit Forum, 2020). The transportation sector is playing a significant role in maintaining the coordinated increase of the modern Chinese industrial system, serving as a vital conduit for the exchange of ideas between domestic and international economic cycles with providing a strong foundation for the stability and security of the country's supply and industrial chains. For the long-term projects and high capital requirements associated with the transportation sector, convertible bonds are an important source of funding for the encouragement and development of associated listed firms. Therefore, this study takes this industry as the anchor for investigation, exploring the application of different models in convertible bond pricing, obtaining the efficiency of valuation and provide empirical recommendation and support for the sustainable development of the economy and society.

In response to the foregoing facts, the significance of this study lies in citing the widely used B-S option pricing model and the binomial option pricing model, which is innovatively developing the binomial model fitting code taking into account the effect of interest rate, dividend payments, resale and redemption clauses on convertible bond pricing in China's convertible bond market in order to correct the deviations. The sample information and data details of this research come from typical convertible bond projects in the transportation industry market in Shanghai stock exchange between 2018 and 2022 to compare the theoretical estimation price and actual price. In the details of the binomial model discussion, this article distinguished the results of innovative conditional fitting under different assumptions to further calculate the deviation rate as well. After all, the conclusion is drawn from judging which of the two models had a lower volatility and better pricing effect.

## 2.0 Literature Review

Theoretical research on the value of convertible bonds began in the 1960s, mainly focusing on defining convertible bonds, ascertaining the conversion price, and roughly describing the value. With the emergence of option pricing theory in the middle of the 1970s, research on convertible bond value likewise rapidly advanced. Presently, scholars worldwide employ diverse techniques to examine convertible bond pricing, broadly classified into three groups: first, establishing the equation's boundary by examining terms that contain rights; and second, generating an approximate return characteristic pricing equation by utilizing the equation and its boundary conditions. The second is using Monte Carlo model to simulate the trajectory of the underlying stock price of the convertible bond and price it by combining the conversion holding value at different stock prices in various nodes.

The structural factor model is the last, primarily covers the application of numerical approaches inside discrete time series, such as the Binomial Tree model and the Black-Scholes model, which offer analytical answers for option pricing over continuous time intervals.

## 2.1 Black-Scholes Model

In the mid-1970s, Louis Bachelier introduced mathematical methods into the study of financial product pricing, which marked the development of option pricing theory. Later, Black Scholes (1973) proposed, for the first time, incorporating the option pricing problem into the continuous-time stochastic process framework and provided a pricing formula for European options in the form of a closed solution. Brennan and Schwartz (1977) subsequently incorporated corporate value into the Black-Scholes pricing model and decomposed the theoretical value of convertible bonds into option value and book value, but ignored corporate dividends. On this basis, Brennan and Schwartz (1980) introduced market interest rate disturbance factors, considered the term structure of interest rates and established a two-factor convertible bond pricing model. This model is more complete than the former, providing more accurate investment forecasts and boundary conditions for the company's optimal strategy. Derman (1996) divided convertible bonds into ordinary options and issuer debt, and proposed a binomial single-factor model to calculate their theoretical value, suppose that uncertainty about the future value of a convertible bond arises from stock price fluctuations. Tsiveriotis et al. (1998) based on the B-S model believe that the value of convertible bonds consists of the conversion option value and book value. The value of the convertible bond is calculated by discounting the risk-free rate, and the value of the option portion is calculated using the B-S model. Carr and Wu (2003) proposed the FMLS model to replace the option pricing model under the traditional B-S erroneous assumption and used the Levy process to replace the Brownian motion describing asset price fluctuations in the B-S model, to better approximate the market reality. In addition to unreasonable assumptions, the Black-Scholes model also has limitations in its use. It can only be applied to the pricing of European options and is not suitable for the pricing of American options. To solve this problem, the academic community has tried to use numerical methods and hybrid methods under the Black-Scholes framework to solve the pricing problem of American options. Ahn and Song (2007) used the trinomial tree option pricing model to conduct pricing research on American options and proved that the model has convergence. Kim et al. (2013) used the iterative method to conduct pricing research on American options. In addition to the above numerical methods, scholars also use hybrid methods to price American options. Fatone et al. (2015) used the Baron-Adsi-Whaley formula to conduct pricing research on American options and summarized the limitations of existing pricing methods.

In a recently published study, Salami (2024) conducted an empirical test of option pricing in markets such as the United States and India. Significant differences between theoretical and actual prices were discovered by the study, indicating that the applicability of the Black-Scholes model in these markets is still limited and that some of its underlying assumptions, such as the normal distribution hypothesis and the volatility constant, may not always be accurately in reality. In order for the model to be more precisely represent market realities, stringent assumptions must be loosened. By adding the jump diffusion model and implied volatility, he improved the BS model's tail distribution features, increasing its statistical significance and pricing accuracy. Du and Chen (2018) computes the theoretical price of convertible bonds in the market using the Black-Scholes PDE formula and compares it to the actual price. The average error between them is 6.61%, showing an approximately well performance in algorithm. After that, the prospect of introducing a stochastic interest rate model into the pricing equation in the future is taken into account. Lin and Zhu (2022) highlight the complexity of pricing operation stems from the complexity of redemption, resale and conversion, which leads to the coexistence of two moving boundaries, depending on the buying price, selling price and conversion ratio. The latest developing B-S model implies that solutions are constrained with two coupled Black-Scholes equations, which discuss so-called finite element methods in Kazbek et al. (2024) published work.

## 2.2 Binomial Tree Related Research

There is a theoretical basis for pricing convertible bonds using the Black-Scholes model and its improved version. However, in practice, considering the impact of multiple factors on the pricing of convertible bonds increases the complexity of convertible bonds. Therefore, some scholars have proposed building a Binomial Tree pricing model based on the B-S model to better cope with this complexity. Cox et al. (1979) first proposed the Binomial Tree option pricing model, which has been widely used in American option pricing. Subsequently, scholars conducted further research and made improvements to the Binomial Tree model. Hull and Nelken (1994) used the Binomial Tree theory and model to consider the risk-free interest rate and the underlying stock price, and proposed a new method to calculate the theoretical value of convertible bonds. Ho and Pfeffer (1996) pointed out that the value of convertible bonds is affected by stock prices and interest rates, and proposed a two-factor Binomial Tree model without arbitrage for pricing. Their findings highlight the importance of stock prices and interest rates on the valuation of convertible bonds. Hung and Wang (2002) independently constructed a Binomial Tree model of interest rates and stock prices, and added a default branch to the two-factor Binomial Tree model to make interest rates and stock prices independent of each other. Gerbessiotis (2003) improved the Binomial Tree model, proposed a trinomial tree option pricing model for the first time, and conducted in-depth research on the pricing of convertible bonds. Altinting and Butler (2005) believe that the conversion price specified by the company is the reasonable purchase price of convertible bonds, which is usually higher than the issuance price, with a certain premium attached. Ren (2009) found through empirical analysis that the difference between the theoretical value calculated using the Binomial Tree model and the actual price is small. Zeng (2013) found through the derivation model that the Binomial Tree model can consider value variables in stages and is better than the finite difference method when dealing with complex clause problems. Dong (2015) studied the convertible bonds of Bank of China and concluded that the estimated value of the Binomial Tree model is more stable than the LSM model. Shvimer and Herbon (2020) use a Binomial Tree option pricing model, taking into account call and put terms, and comparing the theoretical price with the actual market value. The results show that this theory may underestimate the actual value of convertible bonds, and the conclusion is consistent with the general trend of market economy.

In recent years, the research on Binomial Tree pricing principle is still updated. Kai (2018) developed a Binomial Tree model that included liquidity premiums and adjusted it according to different market activity levels to study the impact of market liquidity and trading volume on the pricing of convertible bonds. Liu (2023) used a binomial method to price embedded options and their associated bonds (callable and non-callable) to enable the method to approximate the evolution of short-term interest rates. The results show that the prices of options and two types of bonds are obviously affected by factors and parameters such as bond maturity, coupon rate, redemption price, short-term interest rate volatility and initial short-term interest rate. Dai et al. (2022) proposed a new equity price tree convertible bond pricing model based on the first default model at random interest rates. The model is good for capturing the typical negative (positive) relationship between randomly evolving DP and FV or EP (EPV).

## 2.3 The Pros and Cons Comparison

Black-Scholes (BS) model and Binomial Tree model play a very important role in the subject of convertible option pricing, but at the same time, they also have different limitations to further deepen or refine the pricing field.

First of all, the Black-Scholes model is based on a very solid theoretical foundation and strictly ordered mathematical assumptions, such as geometric Brownian motion and the principle of no-arbitrage pricing, which makes the BS model provide a stable closed solution formula, which is relatively simple to calculate, and is suitable for the pricing of European options. This simplicity has made it widely used in financial markets. But at the same time, BS model assumes too idealized market conditions, such as constant risk-free interest rate, lognormal distribution of stock prices, frictionless market, etc., which is often not true in the actual market (Lee et al., 2023). In addition, there is a volatility smiles phenomenon in the actual market, that is, options with different strike prices have different implied volatility, and the BS model assumes that volatility is a constant, which cannot explain this phenomenon (Batten et al., 2018).

The Binomial Tree model has a completely opposite advantage to the BS model. Constructing a discrete grid of time and price can flexibly deal with various option types, including American options, call options, put options, etc., and complex option terms.

Not only that, the Binomial Tree model can also adjust the dividend profile and volatility changes of the stock as needed, which makes it more practical for dealing with real-world option pricing. In terms of numerical methods, the Binomial Tree model is widely used in a variety of complex financial derivatives obtained by closed solutions, and makes up for the limitations of BS model. However, due to the high complexity of constructing the tree structure, the calculation process is more complicated, especially in the case of more time steps and states, the larger calculation amount limits the calculation accuracy of Binomial Tree fitting. The relatively slow convergence rate slows down the computational efficiency of the Binomial Tree model and brings inconvenience to the research. In addition, the accuracy of the pricing results depends on the selection of time steps, too few steps may lead to inaccurate results, and too many steps will increase the computational complexity (Hull, 2018).

## 3.0 Methodology and Elements Interpretation

This paper mainly uses Matlab to set the asynchronous length and trigger mechanism, runs the loop code to overcome the above defects of the Binomial Tree model, compares the fitting results of different compensation outputs, obtains the general law of pricing accuracy, completes the summative contribution of the model, and improves the application ability of the model.

### 3.1 Based on Binomial Tree Model

Definition of the Binomial Tree model: The establishment of a Binomial Tree option pricing model is contingent upon certain assumptions:

i. Ideal Market Conditions

   The market is assumed to be ideal, characterized by the absence of trading frictions, transaction costs, and taxes.

ii. Long and Short Positions

Market participants are allowed both long and short positions.

iii. Infinite Subdivision of Underlying Asset

The underlying asset is divisible into an infinite number of units, allowing for a continuous and detailed representation.

iv. Known Probabilities of Asset Movement

The direction of the underlying asset's movement is limited to upward and downward, with known probabilities associated with each direction.

v. Risk-Free Borrowing and Lending

Investors have the ability to borrow and lend at a risk-free interest rate.

vi. Investor Risk Neutrality

Investors are assumed to be risk-neutral, implying that the expected return on all securities equals the risk-free interest rate.

The Binomial Tree model assumes that the movement of stock prices is composed of numerous small binomial movements, simulating the fluctuation of stock prices over a certain period. The model first divides the time from the option purchase date to the option expiration date into equal intervals. Within each interval, the stock price $S_0$ can only have two possible changes: an increase to $S_u$ or a decrease to $S_d$. Under these specifications, the Binomial Tree model simulates the trajectory of the underlying asset's price during the option's effective period. Through the corresponding tree branches, the option price at each node can be calculated in reverse.

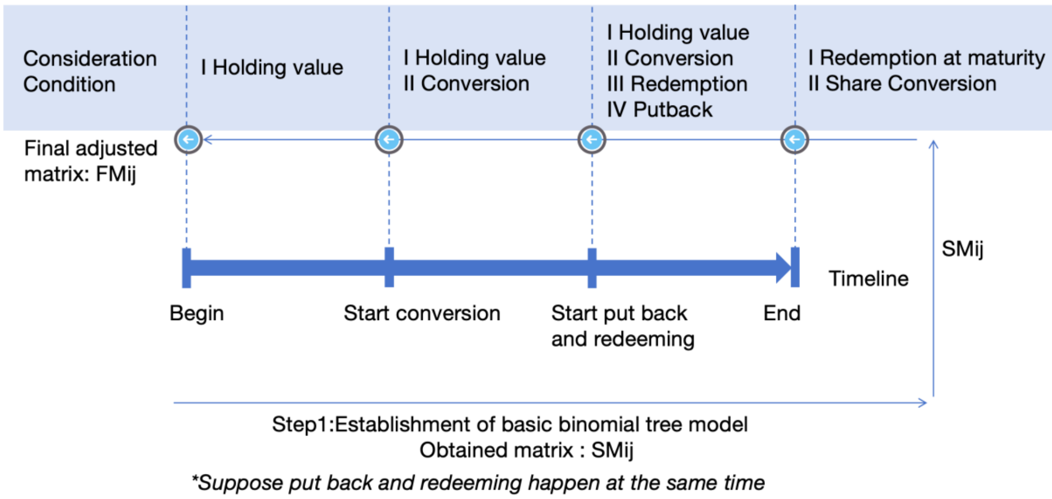


**Figure 1: Binomial Tree Model Flow Diagram**

Using a Single-Period Binomial Tree model as an example, in the scenario of shorting 1 derivative, the investor needs to take a long position in Δ shares of the underlying stock to hedge the risk effectively. This implies that the investor holds an investment portfolio.

In this case, the value of the portfolio is $S_u\Delta$ - $F_u$ when the stock price rises or $S_u\Delta$ - $F_u$ when the stock price falls. To construct a risk-free investment portfolio, this article set $S_u\Delta$ - $F_u$ equal to $S_d\Delta$ - $F_d$. This equation can be solved for Δ, resulting in $\Delta = (F_u - F_d)/(S_u - S_d)$. Here, Δ serves as a hedge parameter, representing the sensitivity of the option's price change to changes in the underlying asset's price.

Assuming that the stock price rises, the current value of the portfolio can be expressed as $(S_u\Delta - F_u)\ e^{-rT}$. Additionally, since the value of the portfolio is also equal to S0Δ - F, this study can derive the pricing formula for the derivative in the Binomial Tree model:

$$F = [\ p * F_u + (1-p) *F_d) * e^{-rT}$$

where p is calculated as $(e^{rT}-d)/(u-d)$. And $S_u=S_0*u$， $S_d=S_0*d$.

After obtaining simulated stock prices at various points, the value of convertible bonds at each node is determined based on the conversion terms, redemption terms, and put option terms. Finally, the initial pricing of convertible bonds at the starting point is deduced using the risk-neutral pricing principle.

Using convertible bonds as the foundation, establish an n-step Binomial Tree model starting with a price S. In this model, the parameters for bond price changes at each time step are defined as follows: $u = e^{(\sigma\sqrt{\Delta t})}$, $d = 1/u$, $p = (a - d)/(u - d)$, where $\Delta t = T/n$.

After obtaining the stock prices for each node (typically denoted as (i, j) in a Binomial Tree model, where (i, j) represents the ith level and jth node in the Binomial Tree), the holding value can be sequentially determined. Conversion value, put option value, and redemption value at different points (i, j) are based on the conversion terms, put option terms, and redemption terms. This process allows to ascertain the value of the convertible bond at that point and ultimately arrive at the initial pricing point.

Let's assume $V_{i,j}$ represents the value of the convertible bond at time (i, j), k is the conversion ratio of the convertible bond, $B_r$ is the redemption price, $B_c$ is the investor's put option value, and $C_{i,j}$ is the holding value at time (i, j). The equation for $C_{i,j}$ can be expressed as follows:

$$C_{i,j} = e^{-r\Delta t}\left(p * V_{i,j+1} + (1 - p) * V_{i+1,j+1}\right)$$

The equation essentially calculates the expected value of the convertible bond at time (i, j) by considering two possibilities: one where the price goes up (p * Vi,j+1) and one where the price goes down ((1 - p) * Vi+1,j+1), both discounted to their present values.

### 3.1.1 Description of Operation Parameters

This study defines this model as an extension matrix to calculate the conversion value of bonds, where i represents the time when each step is binomial, and j represents the possible results of all binomial at time i (from the maximum increase result to the maximum decrease result). Therefore, the share conversion value (sep) at a certain moment is defined as $SM_{ij}$*K (share conversion value * share conversion ratio). Among them, $SM_{ij}$ is deduced by establishing an n-step Binomial Tree model using the benchmark stock price S on the pricing day as the starting point, as shown in the figure. Besides, the redemption and resale conditions stipulate that the execution conditions are x% and y% of the converted value (N), respectively

* [4] $sep = SM_{i,j}\frac{100}{sc}$

**Table 1: Definition of model symbolized parameters**

| | |
|---|---|
| k | Conversion ratio of convertible bonds |
| σ | Annual historical volatility |
| u | Single step up ratio |
| d | Single step down ratio |
| $S_0$ | Current stock price |
| $S_u$ | Single-step stock price increase |
| $S_d$ | Single-step stock price decline |
| $F_u$ | Single step option price increase |
| $F_d$ | Single step options fall in price. |
| Δ | Number of hedged stocks |
| $S_u\Delta - F_u$ | Portfolio price increase |
| $S_d\Delta - F_d$ | Portfolio falling prices |
| r | Risk-free interest rate |
| Δ t | Binomial Tree step size |
| T | Remaining maturity of convertible bonds |
| F | One-step long pricing price |
| P | Next step rising probability |
| $C_{i,j}$ | Holding value of convertible bonds at time i circumstance j |
| $V_{i,j}$ | The value of convertible bonds at time i circumstance j. |
| SMij | Share conversion value at time i circumstance j. |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | 53.31 |
| | | | | | | | | | | | | | 53.16 | 53.01 |
| | | | | | | | | | | | | 53.01 | 52.86 | 52.71 |
| | | | | | | | | | | | 52.86 | 52.71 | 52.56 | 52.41 |
| | | | | | | | | | | 52.71 | 52.56 | 52.41 | 52.27 | 52.12 |
| | | | | | | | | ... | ... | | ... | ... | ... | ... |
| | | | | | | | 23.28 | ... | | 10.25 | 10.23 | 10.20 | 10.17 | 10.14 |
| | | | | | | 23.21 | 23.14 | ... | | 10.20 | 10.17 | 10.14 | 10.11 | 10.08 |
| | | | | | 23.15 | 23.08 | 23.01 | ... | | 10.14 | 10.11 | 10.08 | 10.05 | 10.02 |
| | | | | 23.08 | 23.01 | 22.95 | 22.88 | ... | | 10.08 | 10.05 | 10.02 | 10.00 | 9.97 |
| | | | 23.01 | 22.95 | 22.88 | 22.82 | 22.75 | ... | | 10.02 | 10.00 | 9.97 | 9.94 | 9.91 |
| | | 22.95 | 22.88 | 22.82 | 22.75 | 22.69 | 22.63 | ... | | 9.97 | 9.94 | 9.91 | 9.88 | 9.86 |
| | 22.88 | 22.82 | 22.76 | 22.69 | 22.63 | 22.56 | 22.50 | ... | | 9.91 | 9.88 | 9.86 | 9.83 | 9.80 |
| 22.82 | 22.76 | 22.69 | 22.63 | 22.56 | 22.50 | 22.43 | 22.37 | ... | | 9.86 | 9.83 | 9.80 | 9.77 | 9.74 |

**Figure 2: Take Code 603368 convertible bonds as an example to calculate $SM_{ij}$**

The holding value at a certain moment is defined as $C_{ij}$.

* [5] $C_{i,j}=e^{-r\Delta t}(p*fM_{i+1,j-1}+(1-p)*fM_{i+1,j})$

The actual value of convertible bonds at a certain moment is defined as $fM_{ij}$, and the ultimate goal of the model is to simulate its actual value at $t_0$.

Noted: In code language, coordinate points need to be inverted: fMij=fM（j，i）

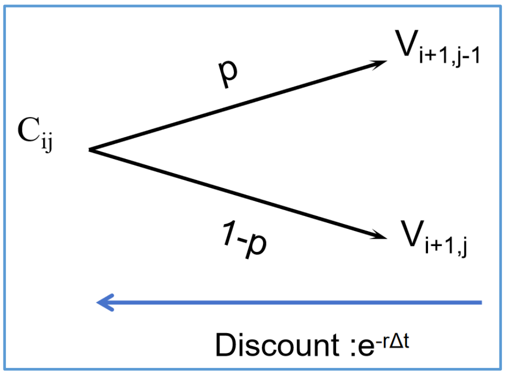


**Figure 3: Binomial Pattern**

### 3.1.2 The following provides an illustration of the procedure and code used to calculate the value of convertible bonds at different times

I. When the convertible bond is at maturity:

At the last moment, convertible bondholders can choose to convert their shares into shares or be redeemed by the issuer. Therefore, investors will choose the one with higher value. Therefore, the final value fMi,j consists of the current conversion value or redemption value EDP (higher) and bond interest. The encode can be presented as:

**fMij = max(Sij ∗ k， Br+$i_n$)**

Ex.

```
if sep < edp+iₙ
fM(j,i)=edp+iₙ;
else
fM(j,i)=sep;
End
```

II. Period *when redemption, putback, conversion and holding of convertible* bonds may *occur* at the same time

**① Redemption or putback**

First of all, This article needs to judge whether the expected conversion value at each moment in this period meets the conditions of conditional redemption and conditional putback. If it does, the convertible corporate bonds will be resold to the company at the face value of the bonds plus the current accrued interest as shown in the company's convertible bond listing announcement:

Ex.

```
if sep<y%N || sep>x%N
fM(j,i)=N+iₙ;
End
```

Secondly, This study is designed to judge which interest payment period is in at this time, so the content can further compile the code and get the value at this time under the condition of conditional redemption or conditional putback:

Ex：

```
if i*tau>=period th
fM(j,i)=N+in;
else i*tau>=(period-a) th
fM(j,i)=100+in-a;
End
```

Therefore, when the expected conversion value of convertible bonds meets the conditions of redemption or putback:

$$fMij = N + i_n$$

**② Do not meet the conditions of redemption and putback.**

In this case, the value of convertible bonds is expressed as the higher of the conversion value and the holding value, that is

$$\mathbf{fMij = max(Sij * k, Cij)}$$

Ex.

```
fM(j,i)=max(100*SM(j,i)/sc,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1)));
```

This article assumes that in the case of share conversion in the current period, the current interest cannot be paid; In the case of continuing to hold convertible bonds, the current interest will be paid normally. Therefore, the code needs to define whether the node time of each step in this time period reaches the annual interest payment node:

Ex.

```
if i*tau=n
fM(j,i)=max(100*SM(j,i)/sc,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+in);
End
```

Therefore, when the expected conversion value of convertible bonds does not meet the conditions of redemption and putback.

$fM_{ij} = \max(S_{ij} * k, C_{ij}+in)$

**in* Only exists when interest is paid annually.

III. When the convertible bond is in the conversion and redemption period but cannot be put back, investors can choose to continue holding, convert, or redeem it.

$V_{i,j} = \max(S_{i,j} * k, B_r, C_{i,j})$

This implies that investors can choose one of three actions, including converting the bond into stock (if the stock price is higher than the conversion price, and the value is $S_{i,j}*k$), redeeming the bond (if the redemption price is higher than the holding value, and the value is Br), or continuing to hold the bond($C_{i,j}$).

IV. When the convertible bond has not yet entered the conversion period, and investors cannot convert, redeem, or put it back, investors can only hold the bond:

$V_{i,j} = C_{i,j}$

This means that investors can only hold the bond, and its value is equal to the holding value $C_{i,j}$.

### 3.2 Based on Black–Scholes Pricing Model

Definition of the Black-Scholes Model: The establishment of a Black-Scholes option pricing model is contingent upon certain assumptions:

1. Short selling of securities is allowed, and the proceeds can be fully utilized.
2. There are no transaction costs or taxes, and all securities can be infinitely subdivided.
3. Stocks do not pay dividends within the option's time frame.

There are no risk-free arbitrage opportunities.

4. Securities trading is continuous.

1. The short-term risk-free interest rate "r" is constant and uniform across all maturities.
2. Stock price movements follow a Brownian motion.

#### 3.2.1 Equation of the Black-Scholes Model

C represents the option price; S represents the stock price; K represents the conversion price; r represents the discount rate; T represents the remaining time to the option's expiration; σ represents the stock price volatility; N(x) is the cumulative distribution function of the standard normal distribution

$C = S_0 * N(d_1) - K * e^{-rT} N(d_2)$

$P = K * e^{-rT} N(-d_2) - S_0 N(-d_1)$

In this equation:

$d_1 = [\ln(S_0/k) + (r + \sigma^2/2)/T]/(\sigma\sqrt{T})$

$d_2 = d_1 - \sigma\sqrt{T}$

According to the above, the pricing model of convertible bonds based on B-S formula adopts the structural idea of decomposing the value of convertible bonds. The following is an analysis of the value of each part:

### 3.2.1.1 Pure Bond Value

The bond portion of the convertible bond, similar to the valuation of regular bonds, can be calculated using cash flow discounting methods. It is calculated as follows:

$$\mathrm{B} = \sum_{i=1}^{n} \frac{I}{(1+i)^n} + \frac{FV}{(1+i)^n}$$

Where: B is Theoretical bond price; i is Annual interest payment on the bond; C is Principal repayment at bond maturity; t is Number of years the bond is held; r is Discount rate.

### 3.2.1.2 Conversion Option Value

The conversion price, conversion time, and conversion price adjustment guidelines are all included in this phrase. Investors may convert convertible bond face value into stocks at the conversion price and then sell them to generate income if the benchmark stock price is higher than the conversion price. Like American call options, investors can choose to convert their convertible bonds into the company's shares. Though early execution is not as profitable as selling call options directly, there is theoretically no example of early execution for American call options. As a result, it can be valued using the Black-Scholes model and considered a European call option.

Formula is used to calculate as:

$$c = S_0\ N(d_1) - K\ e^{-rT} N(d_2)$$

Therefore, the pricing of convertible bonds based on BS model can be expressed as (1)+(2)

## 4.0 Empirical Analysis

### 4.1 Based on Binomial Tree Model

#### 4.1.1 Conditions and Control Variables

In order to verify the measurement of pricing accuracy, this paper collected the transportation industry convertible bonds issued between 2018-2022 on the Stock Exchange market of China (a total of 8 bonds). The specific parameter information is summarized as follows:

**Table 2: Basic data of 8 convertible bonds**

| Code | Maturity | Face value | Share conversion ratio | Stock price | Annualized fluctuation | Initial transfer time | Redemption price at maturity | Conditional redemption price | Sell-back period | Conditional sale price |
|---|---|---|---|---|---|---|---|---|---|---|
| 000582 | 6 | 100 | 8.35 | 8.29 | 0.410518582 | 0.5 | 108 | 130 | 4 | 70 |
| 601006 | 6 | 100 | 7.66 | 6.6 | 0.145314665 | 0.5 | 108 | 120 | 4 | 70 |
| 002930 | 6 | 100 | 20.25 | 18.91 | 0.41986614 | 0.5 | 108 | 130 | 4 | 70 |
| 603535 | 6 | 100 | 22.98 | 19.79 | 0.262989835 | 0.5 | 115 | 130 | 4 | 70 |
| 603713 | 5 | 100 | 134.55 | 132.01 | 0.347744814 | 0.5 | 112 | 130 | 3 | 70 |
| 600029 | 6 | 100 | 6.24 | 6.01 | 0.289247056 | 0.5 | 106.5 | 130 | 4 | 70 |
| 603569 | 6 | 100 | 11.99 | 11.81 | 0.437934727 | 0.5 | 115 | 130 | 4 | 70 |
| 001965 | 6 | 100 | 9.34 | 9.14 | 0.221422247 | 0.5 | 105 | 130 | 4 | 70 |
| | | | | | | | | | | |
| Code | Interest period | IR1 | IR2 | IR3 | IR4 | IR5 | IR6 | Issuance date | Issue price | |
| 000582 | 1 | 0.2 | 0.5 | 1 | 1.5 | 1.8 | 2 | 2021.6.29 | 100 | |
| 601006 | 1 | 0.2 | 0.5 | 1 | 1.8 | 2.6 | 3 | 2020.12.14 | 100 | |
| 002930 | 1 | 0.4 | 0.6 | 1 | 1.5 | 1.8 | 2 | 2020.7.17 | 100 | |
| 603535 | 1 | 0.3 | 0.4 | 0.8 | 1.5 | 2 | 3 | 2022.9.1 | 100 | |
| 603713 | 1 | 0.3 | 0.5 | 1 | 1.5 | 2 | / | 2022.9.16 | 100 | |
| 600029 | 1 | 0.2 | 0.4 | 0.6 | 0.8 | 1.5 | 2 | 2020.10.14 | 100 | |
| 603569 | 1 | 0.5 | 0.8 | 1 | 1.5 | 1.8 | 2 | 2018.11.6 | 100 | |
| 001965 | 1 | 0.1 | 0.3 | 0.6 | 0.8 | 1.5 | 2 | 2019.3.22 | 100 | |

In this case, this study selected S0 to represent the price of each underlying stock at the end of the convertible bond issuance. The actual bond issuance price is 100 CNY, which is the same as the par value. The term of the convertible bonds is six years (especially 603713 takes five years) , and the conversion into stocks begins six months from the date of issuance (t1=0.5) [China's "Regulations on the Administration of Securities Issuance of Listed Companies"], the last two interest-bearing years are the conditional putback period (t2=4). The interest accrual period is one year, and the conditional redemption and call option prices are 130% and 70% of the convertible bond's par price, respectively. r represents the risk-free interest rate of convertible bonds, which takes a value of 2.1%. The paper uses the 5-year government bond interest rate announced by the People's Bank of China on January 1, 2020.

Special note: Collect the closing stock prices of the above eight bond-issuing companies for one year, and use them as a basis to calculate the daily logarithmic return rate and daily standard deviation of each company's stock. The daily standard deviation is then converted to the annual standard deviation as the historical volatility σ（sigma）.

Put the authentic value in prepared code then start to run

① Input variables with different step sizes as controls (50, 100, 200), and multiple simulation results will be obtained to reduce errors.

② Consider all situations in III at the same time to accurately evaluate our results. The code running results are arranged as shown in the following table:

**Table 3: Comparison between Simulated Price *and* Actual Price of Binomial Tree model**

| Code | Step Length | Theoretical value (yuan) | Issue price | difference | Code | Step Length | Theoretical value (yuan) | Issue price | difference |
|---|---|---|---|---|---|---|---|---|---|
| 000582 | 50 | 110.0658 | 100 | 10.0658 | 603713 | 50 | 111.5212 | 100 | 11.5212 |
| | 100 | 111.0347 | 100 | 11.0347 | | 100 | 110.071 | 100 | 10.071 |
| | 200 | 110.3382 | 100 | 10.3382 | | 200 | 110.5869 | 100 | 10.5869 |
| 601006 | 50 | 106.6877 | 100 | 6.6877 | 600029 | 50 | 110.1001 | 100 | 10.1001 |
| | 100 | 106.7443 | 100 | 6.7443 | | 100 | 110.3004 | 100 | 10.3004 |
| | 200 | 106.7661 | 100 | 6.7661 | | 200 | 111.0786 | 100 | 11.0786 |
| 002930 | 50 | 111.039 | 100 | 11.039 | 603569 | 50 | 110.2111 | 100 | 10.2111 |
| | 100 | 109.5884 | 100 | 9.5884 | | 100 | 111.252 | 100 | 11.252 |
| | 200 | 110.1899 | 100 | 10.1899 | | 200 | 110.5188 | 100 | 10.5188 |
| 603535 | 50 | 108.6138 | 100 | 8.6138 | 001965 | 50 | 110.8519 | 100 | 10.8519 |
| | 100 | 108.7718 | 100 | 8.7718 | | 100 | 111.2584 | 100 | 11.2584 |
| | 200 | 108.77 | 100 | 8.77 | | 200 | 110.8764 | 100 | 10.8764 |

### 4.2 Based on Binomial Tree model

Similarly, put the base data in Table 2 into the B-S Model Formula, then get the other results table as follows:

**Table 4: Comparison between Simulated Price and Actual Price of Black–Scholes pricing model**

| Code | Theoretical value (yuan) | Issue price | Difference | Code | Theoretical value (yuan) | Issue price | Difference |
|---|---|---|---|---|---|---|---|
| 000582 | 98.15539604 | 100 | -1.844603958 | 603713 | 138.9814726 | 100 | 38.98147264 |
| 601006 | 97.40824162 | 100 | -2.591758379 | 600029 | 95.13329505 | 100 | -4.866704948 |
| 002930 | 102.7128809 | 100 | 2.712880854 | 603569 | 100.4522104 | 100 | 0.452210368 |
| 603535 | 100.3641591 | 100 | 0.364159138 | 001965 | 95.42287166 | 100 | -4.577128338 |

## 5.0 Finding and Discussion

### 5.1 Result Illustrate

Summarizing the pricing results of the above two models, the conclusion table can be drawn following:



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**Table 5: Summary of simulated price and actual price based on Binomial Tree model and B-S model**

| Code | | Theoretical value (yuan) | Issue price | Difference | Code | | Theoretical value (yuan) | Issue price | Difference |
|---|---|---|---|---|---|---|---|---|---|
| 000582 | 50 | 110.0658 | 100 | 10.0658 | 603713 | 50 | 111.5212 | 100 | 11.5212 |
| | 100 | 111.0347 | 100 | 11.0347 | | 100 | 110.071 | 100 | 10.071 |
| | 200 | 110.3382 | 100 | 10.3382 | | 200 | 110.5869 | 100 | 10.5869 |
| | BSM | 98.15539604 | 100 | -1.844603958 | | BSM | 138.9814726 | 100 | 38.98147264 |
| 601006 | 50 | 106.6877 | 100 | 6.6877 | 600029 | 50 | 110.1001 | 100 | 10.1001 |
| | 100 | 106.7443 | 100 | 6.7443 | | 100 | 110.3004 | 100 | 10.3004 |
| | 200 | 106.7661 | 100 | 6.7661 | | 200 | 111.0786 | 100 | 11.0786 |
| | BSM | 97.40824162 | 100 | -2.591758379 | | BSM | 95.13329505 | 100 | -4.866704948 |
| 002930 | 50 | 111.039 | 100 | 11.039 | 603569 | 50 | 110.2111 | 100 | 10.2111 |
| | 100 | 109.5884 | 100 | 9.5884 | | 100 | 111.252 | 100 | 11.252 |
| | 200 | 110.1899 | 100 | 10.1899 | | 200 | 110.5188 | 100 | 10.5188 |
| | BSM | 102.7128809 | 100 | 2.712880854 | | BSM | 100.4522104 | 100 | 0.452210368 |
| 603535 | 50 | 108.6138 | 100 | 8.6138 | 001965 | 50 | 110.8519 | 100 | 10.8519 |
| | 100 | 108.7718 | 100 | 8.7718 | | 100 | 111.2584 | 100 | 11.2584 |
| | 200 | 108.77 | 100 | 8.77 | | 200 | 110.8764 | 100 | 10.8764 |
| | BSM | 100.3641591 | 100 | 0.364159138 | | BSM | 95.42287166 | 100 | -4.577128338 |

Furthermore, the three-step results of Binomial Tree model operation and the simulation results of BS model are averaged and the volatility is calculated at the same time, and the results are compared in a list:

**Table 6: Simple statistical analysis of simulated pricing**

| | 50 | 100 | 200 | BSM |
|---|---|---|---|---|
| 000582 | 10.0658 | 11.0347 | 10.3382 | -1.844603958 |
| 601006 | 6.6877 | 6.7443 | 6.7661 | -2.591758379 |
| 002930 | 11.039 | 9.5884 | 10.1899 | 2.712880854 |
| 603535 | 8.6138 | 8.7718 | 8.77 | 0.364159138 |
| 603713 | 11.5212 | 10.071 | 10.5869 | 38.98147264 |
| 600029 | 10.1001 | 10.3004 | 11.0786 | -4.866704948 |
| 603569 | 10.2111 | 11.252 | 10.5188 | 0.452210368 |
| 001965 | 10.8519 | 11.2584 | 10.8764 | -4.577128338 |
| Mean | 9.886325 | 9.877625 | 9.8906125 | 3.578815922 |
| Standard deviation | 1.55690731 | 1.534118033 | 1.442754579 | 14.53834018 |
| Mean (Binomial Tree) | 9.884854167 | | | |
| Standard deviation (Binomial Tree) | 1.444838424 | | | |



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Based on the above results, there is a certain error between the simulation using the two models and the actual issue price, but the overall fluctuation is in a gratifyingly small range. Among them, the simulation results based on the Binomial Tree model are generally larger than the actual pricing, while the simulation results based on the BS model are roughly the opposite, which usually floats around the authentic price. Meanwhile, comparing the fitting degree of the two models, there is an obvious rule that the average of the differences based on Binomial Tree model is larger than those based on B-S model. However, the Std presents a totally converse direction.

Consequently, an exact summary can be obtained from the empirical pricing of convertible bonds in the transportation industry market in China, the simulation result of B-S model is more accurate than that of Binomial Tree model, while the stability of Binomial Tree estimation performed better, which means it may be more straightly modify and improve by other compilers systematically. In addition, in the simulation of the pricing of convertible bonds in this industry by Binomial Tree model, different input steps did not have a great impact on the fitting results.

## 5.2 Model Analysis

In the context of the present investigation, it has been ascertained that the employment of two predominant mathematical models has been instrumental in the valuation of convertible bonds within the Chinese transportation sector. Initially, the binomial lattice approach has been utilized to extrapolate the intrinsic values of American-style opening position options, thereby offering a robust framework for the pricing of an array of American options, in addition to extending the applicability to the valuation of convertible securities. Furthermore, the Black-Scholes model, which is extensively integrated into market software for pricing purposes, demonstrates its efficacy through two principal attributes:

- The model’s intrinsic calculations are devoid of errors, ensuring the integrity of the analytical outcomes; and
- The model’s formulae are conducive to direct computational application,

facilitating an expedited and user-friendly valuation process.

It is pertinent to highlight an innovative enhancement to the conventional Binomial Tree model executed within this study. By employing Matlab coding, the research has addressed a multitude of computational challenges, such as the implications of redemption and recovery provisions, along with the precision of interest rate discretization on the integrity of the simulation outcomes. The comprehensive integration of various economic behaviors into the valuation simulation has rendered the model more adaptable and versatile, thereby enhancing its general applicability in the field of convertible bond pricing.

## 6.0 Conclusion

### 6.1 Limitation

The current inquiry, while advancing the field of convertible bond valuation, is unavoidably waiting for improvement in some aspects. Initially, it must be acknowledged that the Binomial Tree model, as a numerical approach, does not provide the same degree of analytical precision as its analytical counterparts. Secondly, despite the enhanced flexibility of the Binomial Tree model over the Black-Scholes model, it still falls short in fully addressing the issue of path dependency inherent in the valuation of convertible bonds. The complex additional provisions of convertible bonds, such as trigger conditions specifying that “the trading price of the underlying stock is below or above m times the conversion price for a consecutive period of n trading days,” present a challenge for the Binomial Tree model. The model’s step size, typically not defined in terms of trading days, hinders the accurate simulation of such path-dependent trigger mechanisms, thus necessitating a simplified consideration of these provisions.

Moreover, the Black-Scholes model exhibits constraints in terms of its fitting capabilities. As a non-node direct analytical method, the BS model is incapable of accounting for the timing and conditions of trigger events associated with convertible bonds, excluding conversion provisions. This limitation extends to the resale and redemption clauses, resulting in a suboptimal fit of the model. Additionally, the BS model is primarily suited for the valuation of European options and American call options, leading to a computational bias in the valuation of American put options, which are often included in the resale clause. This discrepancy introduces an element of inconvenience into the pricing process.

### 6.2 Contribution

This study contributes to a profound comprehension of the pricing mechanisms underlying convertible bonds, elucidating the respective merits and limitations of the Binomial Tree model and the Black-Scholes model in the context of convertible bond pricing simulations. By identifying directions for future research in pricing theory, this investigation holds significant academic value with long-term implications.

The refinement of the pricing model advanced in this study enhances the computability of pertinent factors and diminishes errors, providing an exemplar and strategic direction for the valuation of convertible bonds across diverse market conditions. Moreover, it offers guidance for the pricing of other related corporate convertible bond derivatives, fostering a market economy system characterized by mutual benefit, openness, and transparency between enterprises and investors. Concurrently, it promotes improvements in corporate governance and transparency, reinforcing social equity and justice (SDG 16) and enhancing corporate social responsibility and sustainable management capabilities (SDG 19) (Sdg, 2019).

Beyond these outcomes, the objective and equitable pricing analyses conducted have markedly bolstered investor confidence in convertible bonds, leading to a significant increase in investment within the industry. This support is instrumental in driving the expansion, investment, and innovation of enterprises in the transportation sector, thereby spurring economic growth and generating employment opportunities. Additionally, the industry’s unique characteristics facilitate the allocation of robust financial resources to transportation infrastructure development, enhancing related infrastructural assets and optimizing the efficiency of warehousing and logistics, all of which underpin sustainable urban and community development (SDG 9), as well as industrial innovation and infrastructure construction (Sdg, 2019)

Of greater significance, the social development of transportation enterprises directly stimulates social investment and public welfare, playing a crucial role in bridging the urban-rural divide, transcending geographical limitations, and fostering multidirectional communication. This holds profound importance for China's latest development objectives, including the further alleviation of poverty, the assurance of resident health, the promotion of educational equity, and the achievement of common prosperity.

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## Appendix

Binomial tree running code (taking six-year convertible bonds as an example)

```
%Matlab programming final code
%Part1:data input
S0=xx;%Stock price on the maturity date of the convertible bond issuance
r=xx;%risk free rate
sigma=xx;%volatility of stock
tau=xx;%step length
i1=xx;%First period interest: r1*par value
i2=xx;%Second period interest: r1*par value
i3=xx;%Third period interest: r1*par value
i4=xx;%Fourth period interest: r1*par value
i5=xx;%Fifth period interest: r1*par value
i6=xx;%Sixth period interest: r1*par value
sc=xx;%Conversion price
edp=xx;%redemption price at maturity
%---------------------------------------------
n=6/tau;%Step count
t1=xx/tau;%Start time of stock transfer
t2=xx/tau;%Start redemption time
T=n*tau;%total duration
u=exp(sigma*sqrt(tau));
d=1/u;
a=exp(r*tau);
p=(a-d) / (u-d);%Probability of rising
SM=zeros(n+1);%define matrix
for i=1:n+1
for j=1:i
SM(j,i)=S0*u^(j-1)*d^(i-j);
end
end
SM=flipud(SM);%Matrix transpose
fM=zeros(n+1);
fM(:,end)=SM(:,end)*100 / sc;%max(Conversion price & maturity redemption price)

for i=n+1%Scenario 1: On expiration
for j=n-i+2:n+1
if fM(j,i) < edp
fM(j,i)=edp+i6;
end
end
end
for i=n:-1:t2%Scenario 2: redemption/sellback/conversion/holding occur
for j=n-i+2:n+1
```

```
sep=100*SM(j,i)/sc;
if i*tau==5
if sep<70 || sep>130% Determine whether the conversion price meets the conditions of
redemption and resale
fM(j,i)=100+i5;
else
fM(j,i)=max(sep,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i5);
end
elseif i*tau==4
if sep<70 || sep>130
fM(j,i)=100+i4;
else
fM(j,i)=max(sep,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i4);
end
elseif i*tau==3
if sep<70 || sep>130
fM(j,i)=100+i3;
else
fM(j,i)=max(sep,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i3);
end
elseif i*tau==2
if sep<70 || sep>130
fM(j,i)=100+i2;
else
fM(j,i)=max(sep,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i2);
end
elseif i*tau==1
if sep<70 || sep>130
fM(j,i)=100+i1;
else
fM(j,i)=max(sep,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i1);
end
else
if sep<70 || sep>130
if i*tau>=6% Determine which period the current interest rate is
fM(j,i)=100+i6;
elseif i*tau>=5
fM(j,i)=100+i5;
elseif i*tau>=4
fM(j,i)=100+i4;
elseif i*tau>=3
fM(j,i)=100+i3;
elseif i*tau>=2
fM(j,i)=100+i2;
elseif i*tau>=1
```

```
fM(j,i)=100+i1;
else
fM(j,i)=100;
end
else
fM(j,i)=max(sep,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1)));
end
end
end
end


for i=t2-1:-1:t1%Scenario 3:redemption/conversion/holding occur
for j=n-i+2:n+1
sep=100*SM(j,i) / sc;
if i*tau==5
if sep>130% Determine whether the conversion price meets the redemption situation
fM(j,i)=100+i5;
else
fM(j,i)=max(100*SM(j,i) / sc,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i5);
end
elseif i*tau==4
if sep>130
fM(j,i)=100+i4;
else
fM(j,i)=max(100*SM(j,i) / sc,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i4);
end
elseif i*tau==3
if sep>130
fM(j,i)=100+i3;
else
fM(j,i)=max(100*SM(j,i) / sc,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i3);
end
elseif i*tau==2
if sep>130
fM(j,i)=100+i2;
else
fM(j,i)=max(100*SM(j,i) / sc,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i2);
end
elseif i*tau==1
if sep>130
fM(j,i)=100+i1;
else
fM(j,i)=max(100*SM(j,i) / sc,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i1);
end
```

```
else
if sep>130
if i*tau>=6% Determine which period the current interest rate is
fM(j,i)=100+i6;
elseif i*tau>=5
fM(j,i)=100+i5;
elseif i*tau>=4
fM(j,i)=100+i4;
elseif i*tau>=3
fM(j,i)=100+i3;
elseif i*tau>=2
fM(j,i)=100+i2;
elseif i*tau>=1
fM(j,i)=100+i1;
else
fM(j,i)=100;
end
else
fM(j,i)=max(100*SM(j,i) / sc,exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1)));
end
end
end
end

for i=t1-1:-1:1%Scenario 1: Can only be held
for j=n-i+2:n+1
if i*tau==5
fM(j,i)=exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i5;
elseif i*tau==4
fM(j,i)=exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i4;
elseif i*tau==3
fM(j,i)=exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i3;
elseif i*tau==2
fM(j,i)=exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i2;
elseif i*tau==1
fM(j,i)=exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1))+i1;
else
fM(j,i)=exp(-r*tau)*(p*fM(j-1,i+1)+(1-p)*fM(j,i+1));
end
end
end
disp(fM)
```